# Essex County Math League

May 22, 2024

# GEOMETRY

**DIRECTIONS:** You may write on this test. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect answers will **NOT** lower your score. You may use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score.

**Before the contest starts, fill in question 21 or 22 with the name of your school/town. Fill in the SEAT ID number as stated on the table.**

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

1. A circle has a circumference of $20\pi$. An arc has a length of $\frac{2\pi}{3}$, solve for the measure of that arc.

A. 120
B. 6
C. 36
D. 24
E. NG

2. The measures of the angles of a triangle are $4x + 12$, $5x + 10$, and $7x - 2$. Which of the following best describes the triangle?

A. Acute
B. Right
C. Obtuse
D. Equilateral
E. NG

3. The line $\frac{4x}{9} + 6y = 108$ crosses both axes. Find the sum of its intercepts.

A. 13.5
B. 225
C. 261
D. 4374
E. NG

4. A square is inscribed in a circle of area $49\pi$. Find the perimeter of the square.

A. $14\sqrt{2}$
B. $28\sqrt{2}$
C. $7\sqrt{2}$
D. 98
E. NG

5. Find the area of the circle whose diameter has endpoints $(5, -8)$ and $(25, 10)$.

A. $724\pi$
B. $306\pi$
C. $101\pi$
D. $181\pi$
E. NG

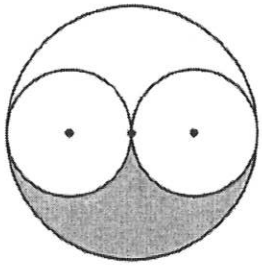
6. The equations of two circles are $8x + x^2 - 2y = 64 - y^2$ and $(x + 12)^2 + (y + 3)^2 = 16$. Find the distance between the two centers.

A. $2\sqrt{10}$
B. $2\sqrt{65}$
C. $2\sqrt{17}$
D. $4\sqrt{17}$
E. NG

7. The volume of a right cylinder is $112\pi$ cubic units. Given the height is 7 units, what is the surface area of the right cylinder?

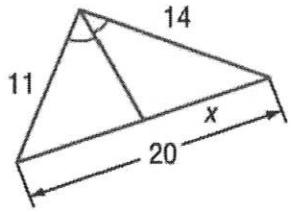
A. $240\pi$
B. $64\pi$
C. $112\pi$
D. $88\pi$
E. NG

8. The three circles are tangent. The diameter of the small circle is 10 cm. Find the area of the shaded region.



A. $200\pi$ cm$^2$
B. $25\pi$ cm$^2$
C. $50\pi$ cm$^2$
D. $100\pi$ cm$^2$
E. NG

9. Find the value of $x$.

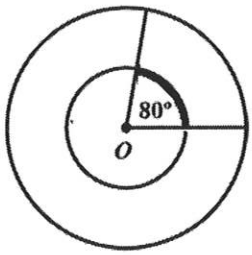



A. $\frac{280}{11}$
B. $\frac{110}{7}$
C. $\frac{56}{5}$
D. $\frac{280}{3}$
E. NG

10. A dilation, centered at $D(-5, -2)$, with a scale factor of $\frac{1}{3}$, is performed on $\Delta ABC$. What would be the new x-coordinate of the dilated image at vertex $B'$, if the triangle preimage had the coordinates, $A(4, 2)$, $B(6, 4)$, and $C(1, 5)$?

A. $\frac{1}{3}$
B. $-2$
C. $-\frac{2}{3}$
D. 0
E. NG

11. Point $O$ is the center of two concentric circles. If the circumference of the large circle is 36 and the radius of the small circle is half of the radius of the large circle, what is the length of the smaller arc of the smaller circle?




A. 10
B. 8
C. 6
D. 4
E. NG

12. An isosceles trapezoid has bases of 20 and 36, along with legs of length 16. Find the area of the trapezoid.

A. $448\sqrt{3}$
B. $224\sqrt{3}$
C. 448
D. 224
E. NG

13. In circle Z, chords $\overline{AB}$ and $\overline{CD}$ intersect at $E$. Let AE be 4 more than EB, EB be 4 more than CE, and CD = 32. Find CE such that AE is 1/3 the length of DE.

A. 2
B. 8
C. 10
D. 16
E. NG

14. Given a trapezoid, if the height is increased by 30% and both bases are decreased by 15%, then the area is _____ .

A. Decreased by 10.5%
B. Increased by 15%
C. Increased by 10.5%
D. Remains the same
E. NG

15. A square and an equilateral triangle have equal perimeters. If the area of the triangle is $10\sqrt{3}$, what is the area of the square?

A. $2\sqrt{10}$
B. $6\sqrt{10}$
C. 22
D. 90
E. NG

16. The equations of two tangent circles are $(x+2)^2+(y+9)^2=25$ and $y^2+4x-20-2y=-x^2$. Which of the following is the point of tangency?

A. $(-2,-4)$
B. $(-2,6)$
C. $(2,-6)$
D. $(-2,4)$
E. NG

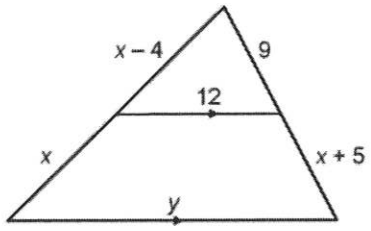
17. If two times the complement of an angle is increased by four times the supplement of the same angle the sum is forty-two more than five times the angle. What is the measure of the angle?

A. 12
B. 78
C. 102
D. 90
E. NG

18. The sum of the interior angles of a regular polygon is 720°. If the apothem of the polygon is 12 cm, find the area of the polygon.

A. $192\sqrt{3}$ cm
B. $48\sqrt{3}$ cm
C. $288\sqrt{3}$ cm
D. $144\sqrt{3}$ cm
E. NG

19. Find the value of y.




A. 10
B. 12
C. 16
D. 32
E. NG

20. A circle is inscribed in a right triangle whose sides are 11, 60, and 61. Find the area of the circle.

A. $4\pi$
B. $9\pi$
C. $16\pi$
D. $25\pi$
E. NG

**Tie Breaker:** *This question must be written on the answer sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

A given parallelogram has sides measuring 7 and 9, and a shorter diagonal measuring 8. Find the measure of the longer diagonal.

ANSWER KEY

1. E
2. A
3. C
4. B
5. D
6. E
7. D
8. B
9. C
10. E
11. D
12. B
13. A
14. C
15. E
16. A
17. B
18. C
19. D
20. D

TB. 14