



Essex County Math League

May 22, 2024

CALCULUS

DIRECTIONS: You may write on this test. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect answers will **NOT** lower your score. You may use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score.

Before the contest starts, fill in question 16 or 17 with the name of your school/town. Fill in the SEAT ID number as stated on the table.

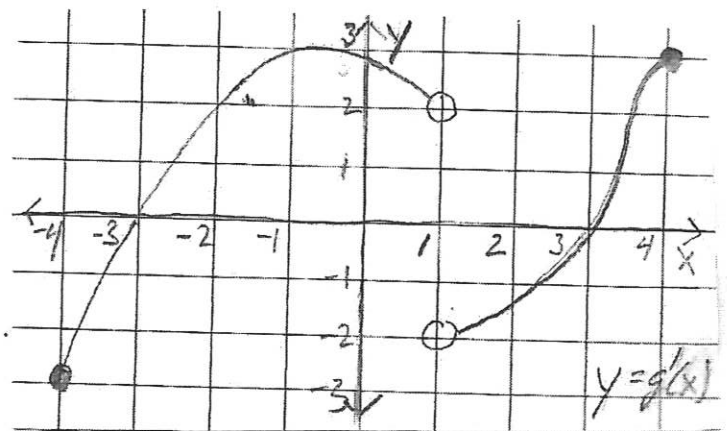
DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

ESSEX COUNTY MATH LEAGUE
CALCULUS
MAY 22, 2024

DIRECTIONS: You may write on this test. Be sure that your name, subject, and school (including town name) are on your scantron sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. NG means the answer is "Not Given," and is an acceptable answer. Problem #16 is an open ended question and should be written on the scantron sheet separately. This question will only be used in case of a tie.

- 1) Let a be a real number. Find $\lim_{x \rightarrow a} \frac{\sqrt{a+6-x} - \sqrt{x-a+6}}{x-a}$.
- A) $-\frac{1}{\sqrt{6}}$ B) $\frac{1}{\sqrt{6}}$ C) $-\frac{1}{\sqrt{a}}$ D) $\frac{1}{\sqrt{a}}$ E) NG
- 2) Let $f(x) = (5x^2 - 4)^7 e^{2x^3}$. The derivative is $f'(x) = p(x)(5x^2 - 4)^6 e^{2x^3}$ where $p(x)$ is a polynomial. Find the sum of the coefficients of $p(x)$.
- A) 8 B) 13 C) 71 D) 76 E) NG
- 3) Consider the curve $x^2y - xy^3 = \sqrt{x} - 2$. Let ℓ be the line tangent to the curve at the point $(4, 2)$. Find the x -coordinate of the intersection of line ℓ with the line $y = 3$.
- A) $\frac{225}{128}$ B) $\frac{380}{63}$ C) $\frac{252}{31}$ D) $\frac{33}{4}$ E) NG

- 4) The function g be a function continuous on $[-4, 4]$. The graph of g' is given to the right. Which of the following, if any, are true on the interval $(-4, 4)$?



- I) g has exactly two critical numbers.
 II) g has a local (relative) maximum value at $x = -1$.
 III) g has exactly two inflection points.
 IV) g is integrable on the interval.
- A) I only B) I, II only C) I, III, IV only D) III, IV only E) NG

- 5) $\int_1^2 \frac{x^3}{\sqrt[3]{3x^2+15}} dx = a + b\sqrt[3]{d}$, where a, b , are rational numbers, and d is a positive cube free integer. Find $a + 2b + 3d$. Note: If the integral is a rational number, let $b = 0$ and $d = 1$.

A) $\frac{77}{12}$ B) $\frac{661}{20}$ C) $\frac{147}{4}$ D) $\frac{81}{2}$ E) NG

- 6) Let b be a positive integer. $\lim_{x \rightarrow -\infty} \frac{\sqrt{bx^2+7}}{5-3x} =$

A) $-\infty$ B) $-\frac{\sqrt{b}}{3}$ C) $\frac{\sqrt{b}}{3}$ D) ∞ E) NG

- 7) Let $F(x) = x \cdot \int_1^{\sqrt{3}x} t^2 \arctan t dt$. Find $F'(1/\sqrt{3})$.

A) $\frac{\pi}{12}$ B) $\frac{\pi}{4\sqrt{3}}$ C) $\frac{\pi}{6}$ D) $\frac{\pi}{4}$ E) NG

- 8) - A sloth is two feet west of a bear stationed at a pole. Both begin moving at the same time. The bear walks south at a rate of 2.25 feet per second, while the sloth crawls towards the pole at a rate of 0.03 feet per second. Find the rate of the change of distance between the sloth and the bear when the bear is 36 feet from the pole. Approximate answer to the nearest thousandth feet per second.

A) 2.247 B) 2.249 C) 2.251 D) 2.253 E) NG

- 9) Let R be the region bounded by $y = 9 - 2x$, $y = mx$, and the y -axis, where m is a positive real number. Find m , if the area of region R is 12.

- A) $\frac{22}{27}$ B) $\frac{11}{8}$ C) $\frac{9 - \sqrt{35}}{2}$ D) $9 - 2\sqrt{6}$ E) NG

- 10) Let $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$. Which of the following statements are true?

- I) f is defined at $x = 0$.
II) f is continuous at $x = 0$.
III) f is differentiable at $x = 0$.

- A) I only B) I, II only C) I, II, III D) II only E) NG

- 11) Let M and m be the maximum and minimum values, respectively, of $f(x) = 2x^3 - 11x^2 - 8x + 5$ on the interval $[-2, 2]$. Find $2M - 3m$.

- A) 39 B) 120 C) $\frac{3503}{27}$ D) $\frac{6419}{27}$ E) NG

12) $\int_0^5 x \cos x \, dx =$

- A) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{5}{n} \cos\left(\frac{k}{n}\right)$ B) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{5}{n} \cos\left(\frac{5k}{n}\right)$ C) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{5k}{n^2} \cos\left(\frac{5k}{n}\right)$ D) $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{25k}{n^2} \cos\left(\frac{5k}{n}\right)$ E) NG

- 13) A student is making a poster, but is restricted to 200 square inches of poster board in the shape of a rectangle. The poster will have a printable area with margins as described: 1) Two inch margins on the top and bottom; 2) One inch margins on each of the left and right sides for the bottom half of the poster; and 3) The margins increase uniformly from one inch on the left and right sides each to two inches on each side going from the middle of the poster to the top of the printable portion of the poster. Find the length of the poster board that will maximize the printable area.

A) $\frac{20\sqrt{6}}{3}$ B) $4\sqrt{10}+4$ C) $8\sqrt{5}$ D) $10\sqrt{6}$ E) NG

- 14) Let $f(x) = \sqrt{x} \ln x$. Find $f'''(1)$.

A) $-\frac{3}{8}$ B) $-\frac{1}{4}$ C) 0 D) $\frac{3}{8}$ E)

- 15) $\int_{-1}^1 \frac{\sin x + 7}{x^2 + 1} dx = a + b\pi$, where a, b are rational numbers. Find $5a - 3b$.

A) $-\frac{33}{2}$ B) $-\frac{27}{12}$ C) $-\frac{21}{2}$ D) undefined E) NG

- 16) Let $g(x) = 3x^4 + 20x^3 - 24x^2 - 354x + 72$. Find the greatest value of a such that $g'(a) = 6$. Final answer must be exact and simplified. Hint: $g'(-3) = 6$.

ANSWER KEY

1. A

2. D

3. C

4. D

5. C

6. C

7. D

8. A

9. B

10. B

11. C

12. D

13. C

14. B

15. C

TB. $-1 + \sqrt{11}$