Essex County Math League

May 22, 2024

ALGEBRA 2

DIRECTIONS: You may write on this test. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect answers will **NOT** lower your score. You may use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score.

Before the contest starts, fill in question 21 or 22 with the name of your school/town. Fill in the SEAT ID number as stated on the table.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

1. Given the functions $f(x) = ax^2 + bx$ and $g(x) = x^2 + 4ax$, where $f(1) = 5$ and $g(2) = 12$. Find the value of $f(-b)$, assuming a and b are constants.

A. -3
B. 1
C. -1
D. 0
E. NG

2. A circle given by the equation $(x-2)^2 + \left(y + \frac{1}{2}\right)^2 = 25$ and a parabola given by the equation $y = x^2 - 8x + 18$ intersect at 2 points. What is the straight-line distance between the vertex of the parabola and the center of the circle?

A. $\frac{13}{2}$
B. $\frac{\sqrt{41}}{2}$
C. $\frac{\sqrt{17}}{2}$
D. $\frac{3}{2}$
E. NG

3. The sum of the roots of $f(x) = ax^2 + 4x + c$ is 8. The values of a and c are real constants. Find the value of $-\frac{1}{2}a$.

A. $\frac{1}{4}$
B. $-\frac{1}{2}$
C. $-\frac{1}{4}$
D. $\frac{1}{2}$
E. NG

4. Let $f(x) = |\sin(x) - x^2 + x| + 2$. What is the range of $f(x)$?

A. $(2, \infty)$
B. $(-\infty, \infty)$
C. $[2, \infty)$
D. $[0, \infty)$
E. NG

5. Which of the following is equivalent to $\frac{1}{2i+x}$, where x is a real number and $i = \sqrt{-1}$.

A. $\frac{2-ix}{x^2-4}$
B. $\frac{2+ix}{x^2-4}$
C. $\frac{2-ix}{x^2+4x}$
D. $\frac{2-ix}{x^2+4}$
E. NG

6. Given $f(x) = \sqrt{x^2-1} - 1$ and $g(x) = \ln x$. What is the value of $[g \circ f \circ g](e)$?

A. 1
B. 0
C. -1
D. e
E. NG

7. Solve $16^{x^2-2x} = 8^{x+1}$

A. $x = \frac{1}{4}, x = -3$
B. $x = -\frac{1}{4}$
C. $x = -\frac{1}{4}, x = 3$
D. $x = -\frac{1}{3}, x = 4$
E. NG

8. Simplify the expression: $\frac{(x^2+x-6)(x^2-3x-4)}{(x^2-6x+8)(x^2+2x-3)}$

A. $\frac{x+3}{x-4}$
B. $\frac{1}{(x-1)(x-2)}$
C. $\frac{x+1}{x-1}$
D. $\frac{(x-1)^2}{x-4}$
E. NG

9. If the average of 3 consecutive even integers is 36, what is the product of the two smallest numbers?

A. 1224
B. 1368
C. 1292
D. 1156
E. NG

10. The system of inequalities $y \leq 2x - 4$ and $2y \geq -\frac{1}{4}x - 4$ have a solution set that lies within which two quadrants?

A. II, IV
B. I, IV
C. II, III
D. III, IV
E. NG

11. y is inversely proportional to x. When x is 6, y is $\frac{1}{3}$. z is directly proportional to the square of x when $x = 2, z = 8$. Assume $x \neq 0$, find z in terms of y.

A. $\frac{8}{y^2}$
B. $8y^2$
C. $8y$
D. $\frac{8}{y}$
E. NG

12. Consider a rectangle with the length being three more than the width. The rectangle's area is 36. What is the length of the rectangle?

A. $3 + \sqrt{17}$
B. $\frac{3}{2}(1 - \sqrt{17})$
C. $\frac{21}{2}$
D. $\frac{3}{2}(1 + \sqrt{17})$
E. NG

13. Consider the following system:

$$\begin{cases} 2x + 4y = 7 \\ 2y = 4kx + 1 \end{cases}$$

Find the value of k so that the lines are perpendicular.

A. $\frac{1}{2}$
B. 1
C. -1
D. $-\frac{1}{2}$
E. NG

14. Consider the following system:

$$\begin{cases} 2x + 3y = 5 \\ 7x + ay = 2 \end{cases}$$

Find the value of a that makes the system have no solutions.

A. $-\frac{21}{2}$
B. $-\frac{14}{3}$
C. $\frac{21}{2}$
D. $\frac{14}{3}$
E. NG

15. Three years ago, Brianna was half as old as John. John is 12 years older than Brianna. What is the sum of John and Brian's current ages?

A. 15
B. 27
C. 42
D. 36
E. NG

16. Consider the equation $2\log(x^a) + \log(x^{2b}) = 3\log(x^c)$ and $x > 0$. For real constants a, b, and c, which of the following must be true?

A. $a + b = 3c$
B. $a + b = \frac{3}{2}c$
C. $a^2 + b = c^3$
D. $a = b + 2$
E. NG

17. Given the data set:

$$\{6, 6, 3, -1, -9, 39\}$$

What is the difference between the median and the mean of this set?

A. $\frac{13}{6}$
B. $\frac{13}{3}$
C. $\frac{4}{3}$
D. $\frac{17}{6}$
E. NG

18. Let $\frac{x^4-2x^3+4x^2+x+1}{x+1} = ax^3 + bx^2 + cx + d + \frac{f}{x+1}$. Find $a + b + c + d + f$.

A. 6
B. 0
C. 7
D. -1
E. NG

19. Find all solutions to the equation: $\sqrt{x} + 2x = 3$

A. $x = 1$
B. $x = \frac{9}{4}$
C. $x = 1, x = \frac{9}{4}$
D. $x = 1, x = -\frac{9}{4}$
E. NG

20. Assume $f(x)$ is a real valued function defined by $f(x) = \frac{\sin x}{x^2+4}$. Where is $f(x)$ undefined?

A. $x = 2$
B. $x = -2$
C. $x = \pm 2$
D. $x = \pi$
E. NG

Tie Breaker: *This question must be written on the answer sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

Sally is playing a game that involves rolling two six-sided dice. She needs to roll at least a sum of 8 to win the game. What is the probability that she wins the game on her next role as a fraction in lowest terms?

ANSWER KEY

1. D
2. B
3. A
4. C
5. D
6. E
7. C
8. C
9. A
10. B
11. A
12. D
13. B
14. C
15. C
16. B
17. D
18. A
19. C
20. E

TB. $\frac{5}{12}$