



Essex County Math League

May 22, 2024

ALGEBRA 1

DIRECTIONS: You may write on this test. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect answers will **NOT** lower your score. You may use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score.

Before the contest starts, fill in question 21 or 22 with the name of your school/town. Fill in the SEAT ID number as stated on the table.

DO NOT TURN THE PAGE UNTIL YOU ARE TOLD TO DO SO

- 1) If a, b, c, d are 4 different numbers, then the line joining $R(-a, b)$ and $S(-c, -d)$ has the same slope as the line joining $T(a, -d)$ and

- A) (c, b)
- B) $(-c, b)$
- C) $(c, -b)$
- D) $(-c, -b)$
- E) NG

- 2) Given that $x^2 + px + 16 = 0$ has two real and distinct solutions, which of the following most accurately describes the range of possible values for p ?

- A) $p > 8$
- B) $p < -8$
- C) $|p| < 8$
- D) $|p| > 8$
- E) NG

- 3) What is the largest power of 5 which divides the product of the first 100 positive integers: $1 \times 2 \times 3 \times \dots \times 98 \times 99 \times 100$?

- A) 5^2
- B) 5^{20}
- C) 5^{24}
- D) 5^{100}
- E) NG

- 4) Find the range of all values of x which satisfy both inequalities:

$$\frac{x-4}{2} + 2 > \frac{x-3}{3}$$

$$\frac{x+1}{3} - \frac{x+4}{2} \geq -\frac{7}{3}$$

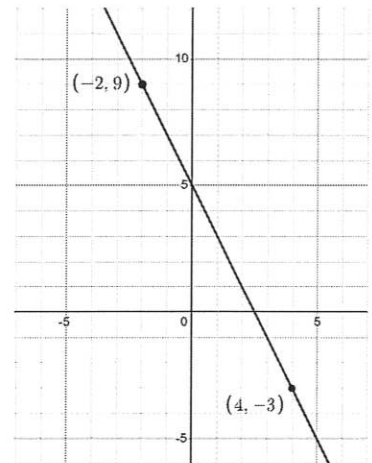
- A) $[-6, 4)$
- B) $[4, \infty)$
- C) $(4, \infty)$
- D) $(-6, 4]$
- E) NG

5) Solve the following equation: $\frac{x^2 - 1}{x - 1} + \frac{x^2 - 6x + 9}{x - 3} = 0$

- A) \emptyset
- B) $x = 1$
- C) $x = -1$
- D) $x = 3$
- E) NG

6) Given the graph of the function $f(x)$ and the equation $g(x) = 2x^2 - 5x + 4$, find the sum of the coefficients of $h(x)$, if $h(x) = f(g(x))$.

- A) 1
- B) 3
- C) -3
- D) -1
- E) NG



7) The expression $E(x) = \frac{3x^2 - 12x + 19}{x^2 - 4x + 5}$ can be written under the form $E(x) = m + \frac{n}{(x-2)^2 + p}$, where m , n , and p are constants. Find the sum of m , n , and p .

- A) 8
- B) 15
- C) 19
- D) 27
- E) NG

8) The ratio between the arithmetic mean of three consecutive integers and the sum of their squares is 1:7. Find the largest of the 3 numbers.

- A) 1
- B) 2
- C) 3
- D) 4
- E) NG

- 9) If the coordinates of the vertices of a parallelogram are $M(-2, 3)$, $N(3, 1)$, $P(-1, 4)$, $Q(-6, 6)$, then one of its diagonals has a slope of:

- A) $-1/3$
- B) $-2/5$
- C) $3/4$
- D) $5/9$
- E) NG

- 10) Find the last digit of $2^{2024} + 3^{2024}$

- A) 1
- B) 3
- C) 5
- D) 7
- E) NG

- 11) Simplify: $\frac{\frac{1}{x^2-1} + (1 + \frac{x-1}{2}) + (\frac{x+1}{2} - 1)}{x^2+2x+2}$

- A) $\frac{-1}{(x+1)(x-1)}$
- B) $\frac{1}{(x+1)(x-1)}$
- C) $\frac{-1}{x^2+2x+2}$
- D) $\frac{(x+1)(x-1)}{x^2+2x+2}$
- E) NG

- 12) If Riley can shovel snow from half of a driveway in 2 hours, and Sam can shovel snow from one quarter of the driveway in 2 hours, how many minutes would it take them to shovel the whole driveway working together at their respective constant rates?

- A) 120
- B) 160
- C) 180
- D) 360
- E) NG

13) Factor Completely: $x^4 - 2x^3 + 2x^2 - 2x + 1$

- A) $x(x^2 + 2)(x - 2) + 1$
- B) $(x^2 - x + 1)(x^2 + x + 1)$
- C) $(x + 1)(x - 1)^3$
- D) $(x^2 + 1)(x - 1)^2$
- E) NG

14) The quadratic function $f(x) = ax^2 + bx + c$ has a vertex at the point $(3, -5)$ and passes through the point $(1, 7)$. Find $b + c - a$.

- A) -2
- B) 0
- C) 1
- D) 2
- E) NG

15) Simplify: $\frac{1 - \frac{3x^2}{1-x^2}}{1 + \left(\frac{1+x}{x}\right)^{-1}}$

- A) 2
- B) $\frac{1-2x}{1-x}$
- C) $\frac{1+x^2}{1-x^2}$
- D) $\frac{x(1-2x)}{1-x^2}$
- E) NG

16) A real estate developer built multiple houses on a new street. Each house needs a number, and for these numbers, special solar lighted digits from 0 to 9 were used. The house numbering starts with 1, with no leading zeros, and altogether, 333 digits were used to number all the houses. Find out how many houses were built.

- A) 111
- B) 144
- C) 147
- D) 150
- E) NG

17) Simplify: $\sqrt{n+1+2\sqrt{n}} - \sqrt{n+1-2\sqrt{n}}$, where n is a positive whole number.

- A) $2\sqrt{n}$
- B) 2
- C) $4\sqrt{n}$
- D) $\sqrt{n+1} - \sqrt{n-1}$
- E) NG

18) Evaluate: $\frac{1}{\sqrt{2}+\sqrt{1}} + \frac{1}{\sqrt{3}+\sqrt{2}} + \frac{1}{\sqrt{4}+\sqrt{3}} + \dots + \frac{1}{\sqrt{99}+\sqrt{98}} + \frac{1}{\sqrt{100}+\sqrt{99}}$

- A) 9
- B) 10
- C) 99
- D) 100
- E) NG

19) Find all values of x that satisfy $|2-x| + |2x+1| \leq 7$

- A) $x \geq -2$ and $x \leq 2$
- B) $x \geq -2$ and $x < 2$
- C) $x \geq -2$ and $x < 3$
- D) $x \geq -2$ and $x \leq 4$
- E) NG

20) Leslie, John and Mike have been helping out in the evenings to process the mail received by an organization. Each evening, Leslie splits the mail received earlier that day in the same order: Leslie, John, Mike, Leslie, John, Mike,..., until all the letters received that day are finished. Knowing that the total number of letters processed by the group throughout their entire volunteering period is 2024 and that Leslie processed 3 letters more than Mike, determine the overall difference between the number of letters processed by John and number of the letters processed by Mike during that period.

- A) 0
- B) 1
- C) 2
- D) 3
- E) NG

Tie Breaker: *This question must be written on the answer sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

An Acela train going from NYC to Washington DC with an average speed of 84 mph left NYC Penn Station at 11:42 am. 50 minutes earlier, an Amtrak regional train left Washington DC heading towards NYC. Knowing that the distance between Washington DC and NYC is 206 miles, and the average speed of the regional train on this route is 54 mph, at what time will the two trains meet?

ANSWER KEY

1. A
 2. D
 3. C
 4. D
 5. A
 6. B
 7. A
 8. C
 9. E
 10. D
 11. B
 12. B
 13. D
 14. C
 15. B
 16. C
 17. B
 18. A
 19. E
 20. C
- TB. 12:52 pm