Essex County Math League

May 24, 2023

GEOMETRY

DIRECTIONS: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

1) Two of the sides of an acute triangle have lengths of 3 cm and 6 cm. To the nearest tenth cm, what is the difference between the longest and shortest possible lengths of the third side.

A) 1
B) 1.5
C) 1.6
D) 1.8
E) NG

2) The perimeter of a triangle is 42 cm. The bisector of one angle divides the opposite side into segments of 4 cm and 8 cm. Find the length, in cm, of the other two sides.

A) 10 and 20
B) 15 and 15
C) 12 and 18
D) 8 and 22
E) NG

3) An isosceles right triangle has a 16 mm hypotenuse. Find the length of the altitude to the hypotenuse, in mm.

A) $4\sqrt{2}$
B) $4\sqrt{3}$
C) 8
D) $8\sqrt{2}$
E) NG

4) The equations of two circles are: $x^2 - 4x + y^2 + 12y - 9 = 0$ and $(x-5)^2 + (y+2)^2 = 9$. Find the distance between the two centers.

A) 3
B) 5
C) 9
D) 18
E) NG

5) Suppose the point (5, –2) is translated to the point (x', y') by the translation $x' = x - 4$ and $y' = y + 3$. This new point is then reflected across the x-axis, translated again by the above transformation, and the reflected across the line y = x. What are its new coordinates?

A) (3, 2)
B) (3, –2)
C) (2, 3)
D) (–2, –3)
E) NG

6) A sidewalk 2 feet wide surrounds a garden 12 by 16 feet. Find the area of the sidewalk in sq. ft.

A) 128
B) 144
C) 192
D) 320
E) NG

7) An isosceles trapezoid with bases 12 and 20 is inscribed in a circle. The center of the circle lies on the longer base of the trapezoid. Find the area of the trapezoid.

A) 120
B) 128
C) 144
D) 256
E) NG

8) Two externally tangent spheres with radii 8 and 12 rest on a table. Find the square of the distance from the center of the larger sphere to the point at which the smaller sphere touches the table.

A) $4\sqrt{15}$
B) $8\sqrt{6}$
C) 20
D) $4\sqrt{38}$
E) NG

9) At 6:20 am, what is the measure of the angle between the hour hand and the minute hand expressed as an angle less than 180 degrees?

A) 60°
B) 70°
C) 80°
D) 120°
E) NG

10) The measures of the angles of a triangle are: $2x + 4$, $8x - 4$, and $12x + 4$. Which of the following best describes the triangle?

A) acute
B) obtuse
C) right
D) isosceles
E) NG

11) Find the measure of an angle such that six times its complement is equal to 20 less than twice its supplement?

A) 22.5°
B) 40°
C) 50°
D) 60°
E) NG

12) A triangle with sides 16, 20, and 32 is similar to a triangle whose shortest side is 4. Find the area of the smaller triangle.

A) $3\sqrt{119}$
B) $6\sqrt{119}$
C) $12\sqrt{119}$
D) $24\sqrt{119}$
E) NG

13) The measures of the angles of a triangle are in the ratio 3:4:5. What is the difference in the measures of the largest and smallest angles?

A) 15
B) 45
C) 60
D) 75
E) NG

14) If the area of a circle is doubled, what is its new circumference?

A) $2\pi r$
B) $2\pi r\sqrt{2}$
C) 4π
D) $4\pi r$
E) NG

15) When the number of sides of a regular polygon is doubled, the measure of each interior angle will increase by only 10 degrees. What is the sum of the interior angles of the original regular polygon?

A) $2520°$
B) $2700°$
C) $5400°$
D) $5760°$
E) NG

16) The distance between the points (4x, y) and (x, –2y) is:

A) $3\sqrt{x^2 + y^2}$
B) $3|x + y|$
C) $\sqrt{3x^2 + 3y^2}$
D) $3|x| + 3|y|$
E) NG

17) In ΔCAT , X is the midpoint of $\overline{CT}$ and Y is the midpoint of $\overline{AC}$. If the area of ΔAXY is 48 sq. cm. less than the area of trapezoid ATXY, what is the area of ΔCAT in sq.cm.?

A) 72
B) 96
C) 108
D) 124
E) NG

18) A chord of length 18 cm is 3 cm from the center of a circle. How long is a different chord in the same circle that is three times as far from the center of the circle as the first chord?

A) 3
B) 6
C) 9
D) 12
E) NG

19) ABCD is a parallelogram. Points E, F, and G are the midpoints of sides $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$ respectively. What is the probability that a point randomly chosen in ABCD is contained in pentagon ABEFG?

A) 0.5
B) 0.625
C) 0.75
D) 0.875
E) NG

20) Which of the following statements must be true if the given statement is true.

If I study hard, I will pass this test.

A) If I do not study hard, I will not pass this test.
B) If I pass this test, then I studies hard.
C) If I do not pass this test, I did not study hard.
D) Passing this test implies that I studied hard.
E) NG

Tie Breaker: *This question must be written on the scantron sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

A lighthouse sits atop a 500 ft tall vertical cliff. From a ship, ¼ mile from the base of the cliff the angle of elevation to the top of the lighthouse is 25.2 degrees. To the nearest foot, what is the height of the lighthouse? **Note:** 1 mile = 5280 feet.

ANSWER KEY

1. B
2. A
3. C
4. B
5. E
6. A
7. C
8. D
9. B
10. B
11. C
12. A
13. E
14. B
15. E
16. A
17. B
18. B
19. C
20. C

TB. 121 feet