Essex County Math League

Wednesday, May 25, 2022

Geometry

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Geometry

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers; incorrect answers will NOT lower your score. You may ONLY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not county as part of the team score. The fifth choice for each question is NG which means "Not Given" and is a valid answer that indicates that the correct answer is not among the answers given.

1. The equations of two tangent circles are $x^2 - 6x = 8y - y^2 - 16$ *and* $(x-3)^2 + (y+2)^2 = 9$.

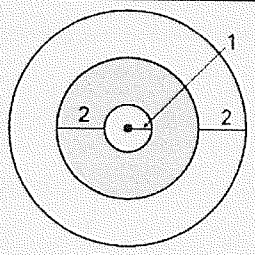
   Which of the following is the point of tangency?

a) (3,-2) b) (3, 1) c) (-3, 2) d) (-3, 1) e) NG

2. Building A is 366 feet tall and Building B is 564 feet tall. If the angle of depression from the top of Building B to the top of Building A is 44°, how far apart are the buildings to the nearest foot?

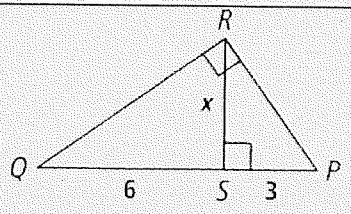
a) 205 feet b) 191 feet c) 285 feet d) 142 feet e) NG

3. Find the probability that a randomly chosen point in the figure lies in the shaded region.



a) 33% b) 29% c) 41% d) 32% e) NG

4. What is the value of x?



a) $3\sqrt{6}$ b) $3\sqrt{3}$ c) $2\sqrt{3}$ d) $6\sqrt{3}$ e) NG

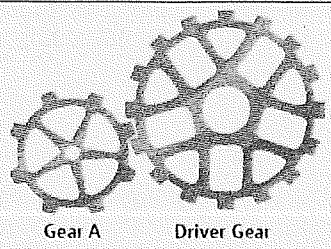
5. $\overrightarrow{YW}$ bisects $\angle XYZ$. Which of the following is an extraneous solution if $m\angle XYZ = (t^2 - 8)°$ and $m\angle XYW = (3t + 4)°$

a) -2 b) -4 c) -8 d) 2 e) NG

6. Pyramid A and Pyramid B are similar. Pyramid A has a volume of 648 m^3 and Pyramid B has a volume of 1,029 m^3. What is the ratio of the surface area of Pyramid A to Pyramid B?

a) 216:343 b) 25:64 c) 36:49 d) 2:3 e) NG

7. Which counterclockwise rotation must the driver gear make for gear A to rotate 90° clockwise?

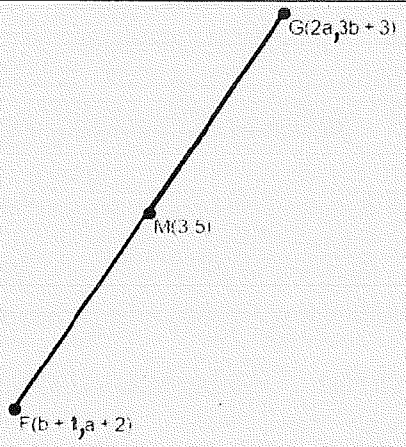


a) 270° b) 45° c) 180° d) 157.5° e) NG

8. The radius of the earth is approximately 4000 miles. If a satellite hovers 400 miles above the earth, approximately what fraction of the Earth is covered by one satellite?

a) $\frac{5}{72}$ b) $\frac{1}{10}$ c) $\frac{5}{36}$ d) $\frac{13}{36}$ e) NG

9. Find the value of the expression a+b using the diagram.

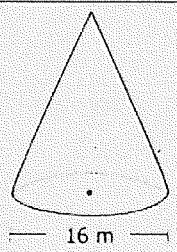


a) 3 b) 4 c) 5 d) 6 e) NG

10. If ABCD is a rhombus, AD = 4x +2, DC = 7x – 13, and BD = 34, find AC.

a) 5 b) 17 c) 22 d) 28 e) NG

11. If the surface area of the cone is $628.32m^2$, find its volume.

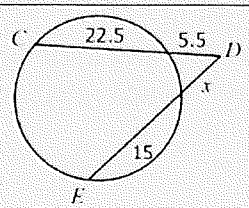


a) $188\pi\ m^3$ b) $261\pi\ m^3$ c) $290\pi\ m^3$ d) $320\pi\ m^3$ e) NG

12. The sides of a triangle have the following lengths $JK = 5x + 17, KL = x - 1$ and $JL = 7x - 9$. Which of the following is a range of possible values of x?

a) $6 < x < 20$ b) $9 < x < 25$ c) $2 < x < 9$ d) $6 < x < 25$ e) NG

13. Find x.



a) 4 b) 5 c) 6 d) 6.5 e) NG

14. Find the area of a right isosceles triangle with a hypotenuse of $20\sqrt{15}$ cm.

a) $1000\ cm^2$ b) $150\sqrt{2}\ cm^2$ c) $1500\ cm^2$ d) $300\sqrt{3}\ cm^2$ e) NG

15. Point A is located at (-4, 9). Find the coordinates of A' if dilated by a scale factor of ¼ with a center of dilation (8, 1).

a) (3,5) b) (-1, 7) c) $(-1, \frac{9}{4})$ d) (2, 4) e) NG

16. For what value of k are the lines $kx - 4y - 4 = 0$ and $3x + (k-1)y - 3 = 0$ perpendicular lines?

a) 4 b) 3 c) -1 d) 6 e) NG

17. The degree measure of an angle in a right triangle is x, and $\cos x = \frac{2}{5}$. Which of the following expressions is also equal to $\frac{2}{5}$?

a) $\sin(x - 45°)$ b) $\sin(90° - x)$ c) $\sin(45° - x)$ d) $\sin(60° - x)$ e) NG

18. A rotation is found by a composite of reflections over a pair of intersecting lines. If the angle formed by the lines is $x°$, the angle of rotation about the point of intersection is 170° what is the value of x?

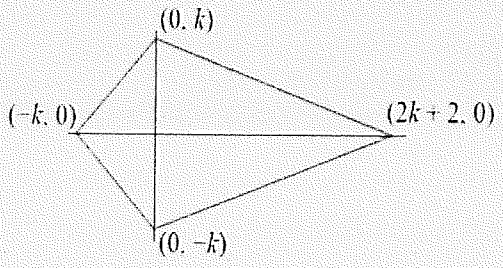
a) 170° b) 60° c) 30° d) 85° e) NG

19. ΔPQR is inscribed in a circle. The ratio of the angle measures of ΔPQR is 4:7:7.

What is the measure of: $\overset{\frown}{QR}$

a) 10° b) 40° c) 80° d) 140° e) NG

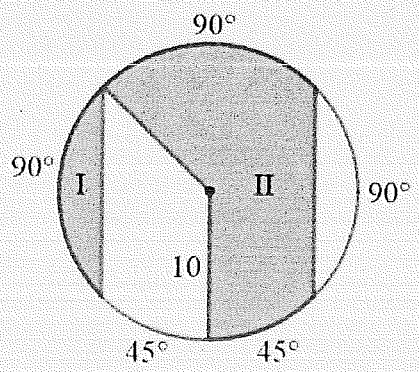
20. If the area of the kite shown is 85 square units, what is the perimeter?



a) $5\sqrt{3} + 20\ units$ b) $10\sqrt{3} + 20\ units$ c) $5\sqrt{2} + 26\ units$ d) $10\sqrt{2} + 26\ units$ e) NG

Tie-Breaker

Find the exact area of the shaded region.



Answer Key

GEOMETRY

1. B

2. A

3. D

4. E – Correct response is $3\sqrt{2}$

5. A

6. C

7. B

8. C

9. A

10. D

11. D

12. B

13. E – Correct response is 7

14. C

15. A

16. A

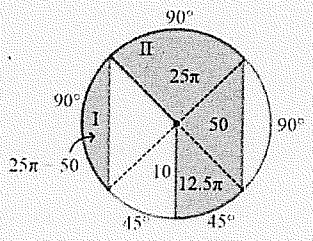
17. B

18. D

19. C

20. D

Tie-Breaker



Area(I + II) = 62.5π u^2