Essex County Math League
May 24, 2018
Geometry

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY only use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

1) Given a right cylinder, if the radius is increased by 20% and the height is decreased by 20%, then its **volume** is _____(to the nearest whole %)

   a) increased by 20%   b) increased by 15%   c) remains the same
   d) decreased by 20%   e) NG

2) The ratio of the consecutive angles of a rhombus is 2:1. If the shorter diagonal is 6cm, what is the **area** of the rhombus?
   a) $18\sqrt{3}cm^2$ b) $18\sqrt{2}cm^2$ c) $18cm^2$ d) $36cm^2$ e) NG

3) The perimeter of a sector of a circle is $\dfrac{10\pi}{3}+12$. What is the **area** of the sector?

   a) $\dfrac{20\pi}{3}$ b) 20π c) 10π d) $\dfrac{40\pi}{3}$ e) NG

4) Two non-intersecting circles have radii of 6 and 9. The common internal tangent is 8. What is the **shortest distance** between a point on one circle and a point on the other circle?

   a) 1   b) 2   c) $\dfrac{7}{3}$ d) 3   e) NG

5)   A non-equilateral triangle has an area of **A** and a perimeter of **P**. If the radius of the circle inscribed in the triangle is **r**, what is the ratio of **A to P**?

a) $\dfrac{r}{2}$ b) $\dfrac{\sqrt{2}}{r}$ c) r   d) $\dfrac{\sqrt{2r}}{r}$ e) NG

6)   The perimeter of a triangle is 30cm. The bisector of one angle divides the opposite side into segments of 4cm and 6cm. Find the length of the other two sides.

a)   6cm, 11cm b) 10cm, 10cm   c)   8cm, 12cm d)   9cm, 11cm e)   NG

7)   The equations of two circles are: $\begin{array}{l} x^2 - 2x + y^2 - 10y + 16 = 0 \\ (x+2)^2 + (y-1)^2 = 9 \end{array}$. Find the distance between their centers.

a)   3   b)   4   c)   5   d) $2\sqrt{3}$ e)   NG

8)   Suppose the point (3, 1) is translated to the point (x', y') by the transformation x'= x + 4, and y' = y - 3. This new point is then reflected across the x-axis, translated again by the above transformation, and then reflected across the line y = x. What are the final coordinates of the point?

a)   (1, 11)   b)   (-1, 11)   c)   (11, 1)   d)   (7, -2)   e)   NG

9)   An equilateral triangle is inscribed in a circle and a second equilateral triangle is circumscribed about the triangle. If the sum of the areas of the two triangles is $60\sqrt{3}$, find the radius of the circle.

a)   1   b)   2   c)   3   d)   4   e)   NG

10)   How many sides does a regular polygon have, if each of its interior angles measures 160°?

a)   9   b)   16   c)   18   d)   24   e)   NG

11) Given triangle ABD with altitude $\overline{AC}$. If $\angle B = 60$ and $\angle D = 50$, which of the following is the correct order of sides?

    a)   AB>AC>AD   b)   AC>AD>AB   c)   AD>AB>AC

    d)   AD>AC>AB   e)   NG

12) Given triangle ABC with points D and E the midpoints of sides AB and AC respectively. If the area of triangle ABC is 36, what is the area of quadrilateral BCED?

    a)   9   b)   18   c) $18\sqrt{2}$ d)   27   e)   NG

13) Two circles with radii of 6 and 2 are externally tangent. A "belt" is wrapped around the exterior of the two circles. What is the minimum length of the "belt?"

    a) $8+12\pi$ b) $\dfrac{4\sqrt{3}+8\pi}{3}$ c) $\dfrac{12\sqrt{2}+4\pi}{3}$ d) $\dfrac{24\sqrt{3}+28\pi}{3}$ e)   NG

14) The equation which represents the set of all points equidistant from the points (5, 4) and (-3, -2) is:

    a) $4x + 3y = 7$ b) $4x - 3y = 7$ c) $3x - 4y = 7$

    d) $3x + 4y = 7$ e)   NG

15) Find the area of a triangle whose coordinates are: (1, 4), (1, 7), (4, 4)

    a) $6+3\sqrt{2}$ b)   4.5   c)   14   d)   9   e)   NG

16) If the distance between the points (a, 3) and (b, 9) is 10, what is the value of $|b - a|$?

    a)   4   b)   8   c)   12   d)   36   e)   NG

17) Find the **volume** of a cylinder whose radius is 5 and whose total surface area is 120π?

    a) 144π b) 175π c) 200π d) 256π e)   NG

18) The two tangent segments to a circle from an external point form an angle of $120°$. If the diameter of the circle is 16, what is the length of each tangent segment?

a) $\dfrac{8\sqrt{3}}{3}$ b)   16   c) $8\sqrt{3}$ d) $\dfrac{16\sqrt{3}}{3}$ e)   NG

19) Given: $\angle 1$ and $\angle 3$ are supplementary. $\angle 2$ and $\angle 3$ are supplementary. If $m\angle 1 = 4x^2 - 2x$ and $m\angle 2 = 4(x^2 + x - 3)$, then $m\angle 3 =$

a) 158°   b)   164°   c)   168°   d)   12°   e)   NG

20) How far above the base of a right circular cone of height h should a plane, parallel to the base, be passed in order to cut off a smaller cone whose volume is one-eighth the volume of the original cone?

a) $\dfrac{h}{8}$ b) $\dfrac{h}{4}$ c) $\dfrac{h}{2}$ d) $\dfrac{2h}{3}$ e)   NG

Tie breaker: This question will be scored only if there is a tie amongst the highest scorers. Please write your answer in the area described by the proctors.

Given right triangle ABC, with $m\angle C = 90$. Segments AX and BY are medians, intersecting at point P. If AC = 6, and BC = 8, find the area of quadrilateral CYPX ?

	ECML Answers 2018							
Algebra 1		**Geometry**		**Algebra 2**		**Advanced Math**	
Quest. #	Answer	Quest. #	Answer	Quest. #	Answer	Quest. #	Answer
1)	C	1)	B	1)	C	1)	D
2)	E	2)	A	2)	D	2)	A
3)	C	3)	C	3)	C	3)	A
4)	B	4)	B	4)	E	4)	B
5)	D	5)	A	5)	A	5)	D
6)	B	6)	C	6)	A	6)	D
7)	C	7)	C	7)	B	7)	C
8)	C	8)	B	8)	A	8)	C
9)	D	9)	D	9)	A	9)	A
10)	A	10)	C	10)	D	10)	A
11)	A	11)	C	11)	C	11)	A
12)	C	12)	D	12)	C	12)	B
13)	E	13)	D	13)	C	13)	B
14)	A	14)	A	14)	B	14)	C
15)	D	15)	B	15)	C	15)	E
16)	B	16)	B	16)	C	TB	2+(1-)=1
17)	B	17)	B	17)	D		
18)	E	18)	A	18)	E		
19)	A	19)	C	19)	C		
20)	A	20)	C	20)	D		
TB	(1, 8)	TB	"8	TB	"1/16"	n	