Essex County Math League

May 24, 2023

CALCULUS

DIRECTIONS: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

1) Let a, b be real numbers. Find $\lim_{x\to\infty}(\sqrt{4x^2+ax+b}-2x)$

A) $\frac{a}{4}$ B) $\frac{a+2b}{4}$ C) $\frac{b}{4}$ D) Does not exist E) NG

2) Let $f(x)=\sqrt[5]{(3x-7)^4}\cdot(2x+5)^7$. The derivative is $f'(x)=\frac{(ax+b)(2x+5)^6}{\sqrt[5]{3x-7}}$. Find $2a-b$.

A) $\frac{443}{5}$ B) $\frac{451}{5}$ C) $\frac{898}{5}$ D) $\frac{906}{5}$ E) NG

3) $\int_2^5 \sqrt{x^2+1}\,dx =$

A) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sqrt{\left(2+\frac{k}{n}\right)^2+1}$ B) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{1}{n}\sqrt{\left(2+\frac{3k}{n}\right)^2+1}$

C) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{3}{n}\sqrt{\left(2+\frac{k}{n}\right)^2+1}$ D) $\lim_{n\to\infty}\sum_{k=1}^{n}\frac{3}{n}\sqrt{\left(2+\frac{3k}{n}\right)^2+1}$ E) NG

4) Let f be a function continuous at all real numbers. Which of the following, if any, are true?

I) If $f'(c)=0$, then f has a local maximum or local minimum at $x=c$.
II) If f has a local maximum or local minimum at $x=c$, then $f'(c)=0$.
III) The global (absolute) maximum and global minimum occurs on any closed and bounded interval.

A) I,II only B) I,II,III C) II only D) III only E) NG

5) Let G be a function such that $G(x)=\int_0^{2x}\sin(t^2)\,dt$. Find $\lim_{x\to 0}\frac{G(x)}{2x^3+x^2+2\cos x-2}=$.

A) $\frac{1}{3}$ B) $\frac{2}{3}$ C) $\frac{4}{3}$ D) Does not exist E) NG

6) Let f,g,h be differentiable functions and let $F(x)=(f\circ gh)(x)$. Use the table below to find $F'(5)$.

x	$f(x)$	$g(x)$	$h(x)$	$f'(x)$	$g'(x)$	$h'(x)$
2	5	6	5	2	3	6
3	6	5	2	3	5	2
5	3	2	3	6	6	3
6	2	3	6	5	2	5

A) 54 B) 90 C) 120 D) 144 E) NG

7) $\int_1^2 2x^5\sqrt{x^2+3}\,dx=a+b\sqrt{d}$, where *a*, *b*, are rational numbers, and *d* is a square free integer. Find *a* + 2*b* + 3*d*. Note: If the integral is a rational number, let *b* = 0 and *d* = 1.

A) $\frac{72}{7}$ B) $\frac{402}{35}$ C) $\frac{2031}{35}$ D) $\frac{1263}{7}$ E) NG

8) Let $f(x)=x(\cos x)^x$. Find $f'(x)$.

A) $(\cos x)^{x-1}(\cos x-x\sin x)$

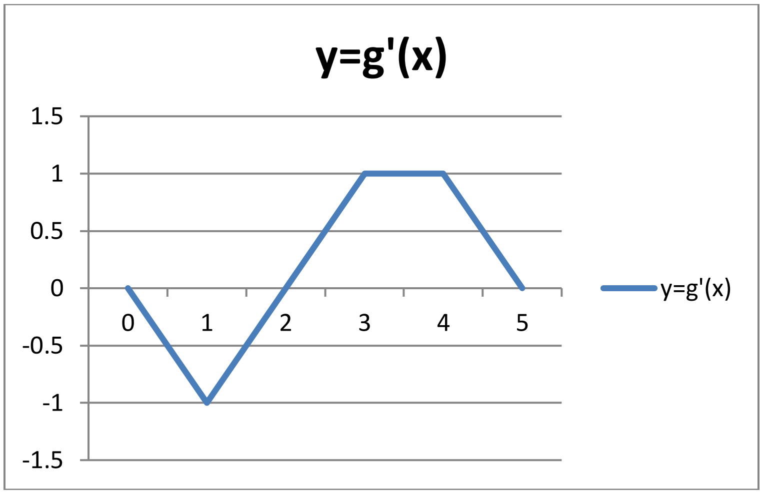
B) $(\cos x)^x(\ln(\cos x)-x\tan x)$

C) $(\cos x)^x(\ln(\cos x)+x-x\ln x\tan x)$

D) $(\cos x)^x(x\ln(\cos x)+1-x^2\tan x)$

E) NG

9) Let g be a function whose derivative is continuous. The graph of g' is given below. Which of the following represents the minimum value of g on [0,5]? Note: $g'(0) = g'(2) = g'(5) = 0$.



A) $g(0)$ B) $g(1)$ C) $g(2)$ D) $g(4)$ E) NG

10) The line tangent to the function $g(x) = x^3 - 3x^2 - 4x + c$ at $x = 3$ is $y = 5x - 18$. Find $g(c-2)$.

A) -183 B) -7 C) 101 D) 177 E) NG

11) A cylindrical with lids on top and bottom is to be formed using a surface area of 100 cm^2 of malleable material. Find the radius, in cm, of either lid that maximizes the volume. Approximate answer to the nearest hundredth.

A) 2.30 B) 3.26 C) 4.89 D) 6.91 E) NG

12) Find $\frac{dy}{dx}$ of the curve $\sqrt{2y} - 10x^3 = 4x^2y^3 - 16$ at the point $(-3, 2)$.

A) $-\frac{540}{863}$ B) $-\frac{539}{864}$ C) $-\frac{156}{863}$ D) $-\frac{155}{864}$ E) NG

13) Let R be the region bounded by the x-axis, $y = x^2$, and the line tangent $y = x^2$ at the point (3,9). Find the area of R.

A) $\frac{9}{8}$ B) $\frac{9}{4}$ C) 9 D) 18 E) NG

14) Let $f(x) = \begin{cases} 6x - x^2, & \text{if } x < 6 \\ 2x - 12, & \text{if } x \geq 6 \end{cases}$. Let k be a real number such that $\int_0^k f(x) = 3\int_0^5 f(x)$. Write k as $a + b\sqrt{d}$, where a, b, are rational numbers, and d is a square free integer. Find $a + b + d$. Note: If c is a rational number, let b = 0 and d = 1.

A) 15 B) 44 C) 100 D) $\frac{7243}{6}$ E) NG

15) Let g be a function such that $g'(x) = \sqrt{x} + \frac{1}{x^2}$ $g(4) = 7$. Find $g(3)$.

A) $\frac{13}{12} + 2\sqrt{3}$ B) $\frac{19}{12} + 2\sqrt{3}$ C) $\frac{37}{6} + \frac{1}{6}\sqrt{3}$ D) $\frac{20}{3} + \frac{1}{6}\sqrt{3}$ E) NG

Tie Breaker: *This question must be written on the scantron sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

A sand dredger dumps sand on a beach at a rate of 1000 ft^3/min into a shape of a right circular cone. The height, h, and the radius, r, of the cone are related by the equation $h = 2r + \frac{5}{r}$. Find the rate of change of the height, in ft/min, when the radius is 4 feet. Approximate answer to the nearest hundredth.