Essex County Math League
May 22, 2019
Calculus

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by the number of correct answers. Incorrect or blank answers will NOT lower your score. You may use only an SAT I approved calculator on this test. NG means the answer is "Not given". This is a viable answer and means that the correct answer is NOT one of the first four listed.

The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors.

The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score.

1) $\lim\limits_{x \to 1} \dfrac{\sin(x^2 - 1) - x^2 + 1}{2x^2 - 5x + 3 + \ln x} =$

A) 0   B) $\dfrac{2}{5}$ C) $\dfrac{2}{3}$ D) does not exist   E) NG

2) Let $f(x) = 5^{x^2 + 1}$. Approximate $f'(1/3)$ to the nearest thousandth.

A) 3.986   B) 5.979   C) 6.415   D) 9.623   E) NG

3) $\int_0^2 \dfrac{1}{\sqrt[3]{4x+1}}\, dx = a + b\sqrt[3]{c}$, where a, b are rational numbers, and c is a positive cube free integer. Find $a + b + c$.

A) $\dfrac{10}{3}$ B) $\dfrac{15}{4}$ C) 6   D) $\dfrac{39}{4}$ E) NG

4) Let f be a function that is defined at $x = a$, such that the derivative is not defined at $x = a$. Which of the following are necessarily true?

I) f is not continuous at $x = a$.
II) There is not a line tangent to f at $x = a$.
III) $f''(a)$ is undefined.

A) I, II only   B) II only   C) II, III only   D) III only   E) NG

5) Let $f(x) = \dfrac{(3x-5)^7}{(4-x)^6}$. Find the sum of the critical numbers for f.

A) $\dfrac{90}{13}$ B) $\dfrac{59}{3}$ C) 22   D) $\dfrac{71}{3}$ E) NG

6) Let $g(x) = \displaystyle\int_0^{3x} \arctan(t^2)\,dt$. Find $g'(\sqrt[4]{3}/3)$.

A) $\dfrac{\pi}{6}$ B) $\dfrac{\pi}{3}$ C) $\dfrac{\pi}{2}$ D) π E) NG

7) Let a be a positive real number. $\displaystyle\lim_{x \to -\infty} \frac{\sqrt{ax^6+3}}{2ax^3-5x} =$

A) $-\dfrac{1}{2\sqrt{a}}$ B) $\dfrac{1}{2a}$ C) $\dfrac{1}{2\sqrt{a}}$ D) ∞ E) NG

8) Let $f(x) = \begin{cases} x^2 \sin(1/x), & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$. Find $f'(0)$

A) 0   B) 1   C) π D) undefined   E) NG

9) A 20 centimeter wire is to be cut into two pieces. One piece will form an equilateral triangle, and the other piece will form a rectangle where the length is twice the width. Let x be the length of the piece to be used for the triangle. Find the value of x that minimizes the sum of the areas of the triangle and rectangle.

A) 0   B) $10\sqrt{3} - 10$ C) 10   D) $80 - 40\sqrt{3}$ E) NG

10) Let $f(x) = \sqrt{x+8} + \ln x$. Let ℓ_1 be the line tangent to f at $x = 1$, let ℓ_2 be the line with equation $2x + 3y = 5$, and let (a,b) be the point of intersection of these two lines. Find a/b.

A) $-\dfrac{7}{13}$ B) $-\dfrac{1}{19}$ C) $\dfrac{3}{13}$ D) $\dfrac{4}{9}$ E) NG

11) Let R be the region in the first quadrant bounded by $y=10-x^2$, $y=|x-2|$, and the axes. Find the area of R.

A) $\dfrac{37}{2}$ 　　　　 B) 19 　　　　 C) $\dfrac{41}{2}$ 　　　　 D) $\dfrac{45}{2}$ 　　　　 E) NG

12) Let $g(x)=xe^{x^2-1}$. Find $g''(1)$.

B) 3 　　　　 B) 4 　　　　 C) 8 　　　　 D) 10 　　　　 E) NG

13) Let f be a continuous function on a closed, bounded interval $[a,b]$. Which of the following statements are necessarily true?

I)　 f is differentiable on $[a,b]$.
II)　 f is Riemann integrable on $[a,b]$.
III)　 f has a maximum value on $[a,b]$.
IV)　 $\dfrac{d}{dx}\int_a^x f(t)dt = f(x)$ for $a<x<b$.

A) I, II only 　　　 B) I, II, III only 　　　 C) I, II, IV only 　　　 D) II, III, IV only 　　　 E) NG

14) $\int_2^3 \frac{x}{1-x^2}\,dx =$

A) $\ln \frac{\sqrt{6}}{4}$ B) $\ln \frac{2\sqrt{6}}{3}$ C) $\ln \frac{8}{3}$ D) undefined   E) NG

15) Let $f(x)=(2x)^{5x}$. Then $f'(x)=(2x)^{5x}(a+b\ln x)$ where a, b are real numbers. Find $a+b$.

A) $\frac{15}{2}+\ln 25$ B) $\frac{15}{2}+\ln 32$ C) $10+\ln 25$ D) $10+\ln 32$ E) NG

TIE BREAKER

16) Xavier is standing 5 feet east of Yolanda when they both start walking. Xavier walks west at a rate of 2.3 feet per second while Yolanda walks south at a rate of 2.5 feet per second. Find the rate that the distance between Xavier and Yolanda is changing when Yolanda is 14 feet from her original location. Write the final answer to the nearest hundredth feet per second.

Solutions

1. A
2. C
3. B
4. D
5. B
6. D
7. A
8. A
9. D
10. B
11. A
12. D
13. D
14. A
15. D
16. 3.31