

ESSEX COUNTY MATH LEAGUE
CALCULUS
MAY 24, 2018

DIRECTIONS: You may write on this test. Be sure that your name, subject, and school (including town name) are on your scantron sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. NG means the answer is "Not Given," and is an acceptable answer. Problem #16 is an open ended question and should be written on the scantron sheet separately. This question will only be used in case of a tie.

- 1) $\lim_{x \rightarrow 0} \sqrt{x} \sin \frac{2}{x} =$
- A) 0 B) 1 C) 2 D) ∞ E) NG
- 2) Let $f(x) = (x^6 + 1)\arctan(x^3)$. Find $f'(-1) =$
- A) $\frac{2-3\pi}{2}$ B) $\frac{3\pi-6}{2}$ C) $\frac{2+3\pi}{2}$ D) $\frac{6+3\pi}{2}$ E) NG
- 3) Let c be a constant. $\lim_{x \rightarrow -\infty} \frac{(x^2 + c)(3x - 5)^2}{2x(7x^3 - 12)} = \frac{a}{b}$, reduced, where a and b are integers, with $a > 0$. Note: If limit is an integer, let $b = 1$. Find $b - 2a$.
- A) -4 B) 1 C) 8 D) limit does not exist E) NG
- 4) Let f be a continuous function on the set of real numbers. Which of the following statements, if any, are true?
- I) Local extreme values only occur at values c , where $f'(c) = 0$.
II) If $f'(c) = 0$, then a local extreme value occurs at c .
III) f has an absolute maximum value on the interval $[1, 4]$.
- A) I only B) I, II only C) I, II, III D) I, III only E) NG

5) Let $f(x) = \frac{e^{x^2}(2x-5)^5}{(4-x^2)^7}$. The derivative $f'(x) = \frac{e^{x^2}(2x-5)^4(p(x))}{(4-x^2)^8}$ where p is a polynomial function.

Find the sum of the coefficients of $p(x)$.

- A) -30 B) -29 C) -21 D) -9 E) NG

6) $\int_0^4 |6-x^2| dx =$

- A) $4\sqrt{6} - \frac{8}{3}$ B) $4\sqrt{6} + \frac{8}{3}$ C) $8\sqrt{6} - \frac{8}{3}$ D) $\frac{136}{3}$ E) NG

7) Let g be a function whose derivatives all exist at $x = 0$, and suppose $g^{(k)}(0) = k$. $\lim_{x \rightarrow 0} \frac{xg(x^2)}{x^3 - x + \sin x} = \frac{a}{b}$, reduced, where a and b are integers, with $a > 0$. Note: If limit is an integer, let $b = 1$. Find $b^2 - 4a$.

- A) limit does not exist B) -4 C) -3 D) 1 E) NG

8) The hypotenuse of a right triangle is increasing at a rate of 30 cm/min. The side opposite a non-right angle α remains constant at 20 cm. Find the rate that α is changing when the hypotenuse is 50 cm. Approximate answer to the nearest hundredth radians per minute.

- A) -11.78 B) -2.38 C) -0.60 D) -0.26 E) NG

9) Let g be a function such that $g'(x) = (x^2 - x - 2)(x^2 - 6x + 8)$. Find the sum of all x -values that correspond to local extreme values.

A) 3

B) 5

C) 7

D) 9

E) NG

10) Let $h(x) = e^x \int_0^{3x} e^t dt$. Find $h'(0)$.

A) e

B) 3

C) $3e$

D) Not enough info. E) NG

11) $\int_0^1 \frac{x^5 + x^2}{x^6 + 1} dx =$

A) $\frac{1}{24}(\ln 16 + \pi)$

B) $\frac{1}{12}(\ln 4 + \pi)$

C) $\frac{1}{12}(\ln 4 + 3\pi)$

D) $\frac{1}{4}(\ln 16 + \pi)$

E) NG

12) Let $f(x) = 3x^5 - 15x^4 - 50x^3 + 450x^2 + 400x - 1760$. Find the sum of the squares of the x -coordinates of the inflection points.

A) 9

B) 10

C) 14

D) 19

E) NG

- 13) A poster board is to have a rectangular writing space of 100 square inches with one inch margins on the sides, two inch margin on top, and a three inch margin on the bottom. Find the minimum possible area of the poster board. Approximate final answer to the nearest hundredth.

A) 113.16 B) 141.62 C) 157.43 D) 173.25 E) NG

- 14) Let f be a function whose second derivative exists everywhere on a closed and bounded interval $[a, b]$. Which of the following statements, if any, are true?

I) f has a maximum and minimum value on $[a, b]$.

II) f' is bounded on $[a, b]$.

III) If N is between $f'(a)$ and $f'(b)$, there exists c , where $a < c < b$ such that $f'(c) = N$.

IV) There exists c , where $a < c < b$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

B) I only B) I, II only C) I, II, III, IV D) I, II, IV only E) NG

- 15) Let $g(x) = \begin{cases} x^2 + ax + b/x, & \text{for } x < -2 \\ ax^2 - bx, & \text{for } x \geq -2 \end{cases}$. Let a and b be real numbers that make f continuous and differentiable everywhere. The sum $a + b =$

A) $\frac{3}{8}$ B) $\frac{2}{3}$ C) $\frac{4}{5}$ D) $\frac{16}{3}$ E) NG

TIE BREAKER

- 16) An ellipse centered at the origin contains the points $(0, \pm 7)$ and has foci $(\pm 3, 0)$. Find the slope of the line tangent to the ellipse in the fourth quadrant at the point $x = 2$. Approximate final answer to the nearest hundredth.

ECML 2018
Calculus Answers

1. A
2. D
3. A
4. E
5. A
6. C
7. D
8. D
9. A
10. B
11. B
12. D
13. D
14. C
15. A
16. 0.25