Essex County Math League

May 24, 2023

ALGEBRA 2

DIRECTIONS: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

1)   Find the sum of the solution(s) of the equation: $\frac{x}{x+4} = \frac{16-4x}{x^2-16}$.

A) –4
B) 0
C) 4
D) 8
E) NG

2)   Let $f(x)$ be a function with $f(4) = -1$. Let $g(x) = -x^2 + bx - 4$ be the inverse of $f(x)$, (for appropriate values of x). Find the value of b.

A) –9
B) –7
C) 3
D) 9
E) NG

3)   Find the equation for the sets of points that are equidistant from the points $(5, -2)$ and $(-1, 2)$.

A) $2x + 3y = 4$
B) $2x - 3y = 4$
C) $3x - 2y = 6$
D) $3x + 2y = 6$
E) NG

4)   Let m be an integer. When the expression $\frac{m+i}{5+3i}$ is written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$, what is the sum of a and b.

A) $\frac{m+4}{17}$

B) $\frac{m+4}{16}$

C) $\frac{2m-5}{16}$

D) $\frac{m-4}{17}$

E) NG

5)   Solve for x: $\left(\frac{3}{4}\right)^{3x-7} = \left(\frac{4}{3}\right)^{2x+5}$

A) $\frac{-12}{5}$

B) $\frac{2}{5}$

C) $\frac{12}{5}$

D) 12

E) NG

6)   Find the multiplicative inverse of the sum of the solutions of the equation:
$$x + \sqrt{14 - x} = 2$$

A) –3

B) $-\frac{1}{3}$

C) $\frac{1}{3}$

D) 3

E) NG

7)   Simplify: $25^{\log_5 3 + 2\log_5 x}$

A) $(3 + 2x)^2$

B) $(3 + 2x)^5$

C) $3x^4$

D) $9x^4$

E) NG

8)   Let $f(x)$ and $g(x)$ be inverse function of each other. Let $f(x) = x^2 - 6x + c$ with $x \le 3$. If $f(x)$ has a vertex at $(3, 0)$, then $g(x) =$ _____.

A) $\sqrt{x} - 3$

B) $3 - \sqrt{x}$

C) $3 + \sqrt{x}$

D) $\sqrt{x} + 3$

E) NG

9)   If $x = -\frac{1}{2}$ is a root of $2x^3 - x^2 - 13x - 6 = 0$, find the other two roots.

A) –2 and –3

B) –2 and 3

C) 2 and –3

D) 2 and 3

E) NG

10) The 5 starting players on the girls' basketball team always give each other a "high five" after they win a game. How many "high fives" do they exchange after a win?

    A) 5
    B) 10
    C) 15
    D) 20
    E) NG

11) A quadratic equation has roots of $3 \pm \sqrt{5}$. If the smaller root is set equal to a, the larger root can be written as:

    A) $-a$
    B) $-a\sqrt{5}$
    C) $a + \sqrt{5}$
    D) $6 - a$
    E) NG

12) If the domain of $f(x) = x^2 - 2x - 8$ is restricted to $x^2 < 9$, the range is given by:

    A) $-9 < f(x) < 7$
    B) $-8 < f(x) < 7$
    C) $-9 < f(x) < 0$
    D) $-2 < f(x) < 4$
    E) NG

13) Find the distance between the points of intersection of the given line and circle:

$$x - 7y + 25 = 0 \text{ and } x^2 + y^2 - 25 = 0$$

    A) $2\sqrt{5}$
    B) $5\sqrt{2}$
    C) $2\sqrt{7}$
    D) $7\sqrt{2}$
    E) NG

14) Solve for x: $8^{x^2-4} = \dfrac{1}{64}$

    A) $\sqrt{2}$
    B) $\pm\sqrt{2}$
    C) $2\sqrt{2}$
    D) $\pm 2\sqrt{2}$
    E) NG

15) Solve for x: $|3x - 2| = |2x + 1|$, where $x \geq \frac{1}{2}$.

    A) $\frac{1}{2}$
    B) $\frac{2}{3}$
    C) 2
    D) 3
    E) NG

16) There are approximately 3000 miles between LA and NYC. On a particular day, there is a 100-mph wind blowing West to East. A plane leaves LA at 10:00 am and arrives in NYC at 3:00 pm while a similar plane leaves NYC at the same time headed towards LA. At what time does the second plane arrive in LA?

    A) 3:00 pm
    B) 4:00 pm
    C) 5:00 pm
    D) 6:00 pm
    E) NG

17) If $f(x) = 1 - f(x - 1)$, express $f(x + 1)$ in terms of $f(x - 1)$.

    A) $f(x - 1)$
    B) $1 + f(x - 1)$
    C) $1 - f(x - 1)$
    D) $f(x - 1) - 1$
    E) NG

18) Given: $4x^2 + 5x - 6 = 0$, find the product of the sum and the product of the roots.

    A) $\frac{-3}{2}$
    B) $\frac{-5}{4}$
    C) $\frac{3}{2}$
    D) $\frac{5}{4}$
    E) NG

19) Find the polynomial equation of lowest degree having integer coefficients whose roots are –4 and 3 + 2i.

A) $x^2 + (1 - 2i)x - (12 + 8i) = 0$
B) $x^3 + 4x^2 - 9x - 36 = 0$
C) $x^3 - 2x^2 - 11x + 52 = 0$
D) $x^3 + 4x^2 + 9x + 36 = 0$
E) NG

20) A backyard pool has 2 inlet water valves (A and B) and 1 outlet valve (C). If only valve A is opened, the pool will fill in 10 hours. If only valve B is opened, the pool will fill in 12 hours. If the pool is filled and all 3 valves are opened at once, the pool will be empty in 60 hours. How long will it take for valve C to empty a full pool if the other two valves are not opened?

A) 5 hrs.
B) 6 hrs.
C) 10 hrs.
D) 30 hrs.
E) NG

Tie Breaker: *This question must be written on the scantron sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

A tetrahedral die has its 4 faces labeled 1, 2, 3, and 5. The die is rolled twice and the sum of its two downside faces is found. What is the probability that the sum is six?

ANSWER KEY

1.   E
2.   A
3.   C
4.   A
5.   B
6.   C
7.   D
8.   B
9.   B
10.   B
11.   D
12.   A
13.   B
14.   B
15.   D
16.   E
17.   A
18.   E
19.   C
20.   B
TB.   3/16