# Essex County Math League

## Wednesday, May 25, 2022

# Algebra II

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers; incorrect answers will NOT lower your score. You may ONLY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is NG which means "Not Given" and is a valid answer that indicates that the correct answer is not among the answers given.

1. Solve:

$$2 < \frac{x+6}{2} < 5 \text{ and } (4 - x > 5 \text{ or } 4 + x > 7)$$

A.) $(-1, 4)$     B.) $(-2, -1) \cup (3, 4)$     C.) $(-\infty, \infty)$          D.) $\emptyset$          E.) NG

2. Given that a is a positive constant, solve the following equation:
$$a^{2x} - 10a^x + 9 = 0$$

A.) 0          B.) $0, \frac{\log 9}{2\log a}$          C.) $0, \frac{\log 9}{\log a}$          D.) $0, \frac{\log a}{\log 9}$     E.) NG

3. A car radiator is completely filled with 5 L of 30% antifreeze solution. How many liters must be drawn off and replaced by 80% antifreeze solution to leave the radiator filled with 60% antifreeze solution?

A.) 7.5 L          B.) 3 L          C.) 2 L          D.) 0.5 L          E.) NG

4. Simplify:

$$(a^4 + a^2b^2 + b^4) \div (a^6 - b^6) \cdot (a + b)$$

A.) $\frac{1}{a-b}, a \neq -b$          B.) $\frac{1}{a+b}, a \neq b$          C.) $\frac{(a^4+a^2b^2+b^4)(a+b)}{(a-b)(a^4+2a^2b^2+b^4)}$

D.) $\frac{(a^4+a^2b^2+b^4)}{(a-b)(a+b)^2}$          E.) NG

5. Given $f(x) = \frac{1-x}{x}$, express $\frac{f(x+h)-f(x)}{h}$ as a single simplified fraction.

A.) $-1$

B.) $-\frac{1}{x(x+h)}, h \neq 0$

C.) $\frac{-x^2-x+h}{hx(x+h)}$

D.) $\frac{x^2-x-h+2hx}{hx(x+h)}$

E.) NG

6. Solve for x. (a, b, and c are positive integers)
$$\log_a(\log_b(\log_c x)) = 0$$

A.) $c^b$

B.) $c$

C.) $1$

D.) $b^a$

E.) NG

7. Oh no! You forgot your locker combination! You remember that there are three different numbers in the combination. The lock's dial shows numbers from 0 to 35. If you would test one combination every 12 seconds, how long would it take to test all the possible locker combinations consisting of 3 different numbers?

A.). 59.5 min

B.) 59.5 hr

C.) 142.8 hr

D.) 8568 hr

E.) NG

8. Find the term in the expansion of $(4y - 3y^{-2})^{12}$ that does not contain y.

A.) -5308416

B.) 5308416

C.) 495

D.) 2627665920

E.) NG

9. Divide:
$$\frac{x^8 - a^8}{x^3 - ax^2 + a^2x - a^3}$$

A.) $x^5 - a^5$

B.) $\frac{x^2-a^2}{(-ax^2+a^2)}$

C.) $x^5 - ax^4 - a^4x + a^5$

D.) $x^5 + ax^4 + a^4x + a^5$

E.) NG

10. A canoeist (who paddles at a constant speed) has paddled upstream for two miles from her camp when she passes a log being carried along by the river's current. She paddles upstream for one more hour before turning around and paddling back to her camp. She reaches her camp at the same moment as the log. What is the speed of the current?

A.) 1 mph

B.) $\frac{1}{2}$ mph

C.) 2 mph

D.) cannot be determined from the given information

E.) NG

11. The load that a beam with a rectangular cross section can support is jointly proportional to the beam's width and the square of its depth and inversely proportional to its length. Beam A is half as wide, twice as long, and three times as deep as Beam B. If $L_A$ is the load that Beam A can support and $L_B$ is the load that Beam B can support, what is the ratio of $L_A$ to $L_B$?

A.) $\frac{3}{4}$

B.) $\frac{9}{4}$

C.) 3

D.) 9

E.) NG

12. Assuming that all variable exponents are positive integers, simplify:

$$x^{a-b}(x^{a+b} - x^{b-a} + x^b)$$

A.) $x^{a^2-b^2} - x^{-(a^2-2ab+b^2)} + x^{ab-b^2}$

B.) $x^{2a} + x^a$

C.) $x^{2a} + x^a + 1$

D.) $x^{2a} + x^a - 1$

E.) NG

13. A swimming pool is 5 m longer than it is wide and is surrounded by a deck 2 m wide. The area of the pool and deck together is 140 square meters greater than the area of the pool alone. What are the dimensions of the pool?

A.) 9 m by 14 m

B.) 13 m by 18 m

C.) 17 m by 22 m

D.) 31.5 m by 36.5 m

E.) NG

14. Given $f(x) = \frac{x}{x^2-4}$ and $g(x) = \frac{1}{x+1}$, what is the domain of $f(g(x))$ ?

A.) $\mathbb{R}\ except - 2, -1, -\frac{3}{2}, -\frac{1}{2}, 2$    B.) $\mathbb{R}\ except - 1, -\frac{3}{2}, -\frac{1}{2}$    C.) $\mathbb{R}\ except - 2, 2$

D.) $\mathbb{R}\ except - 2, -1, 2$    E.) NG

15. A 15-member special committee met three times. The average attendance at the meetings was 9 people. Twice as many members were present at the third meeting as the first. What is the sum of the possible solutions to how many people were present at the third meeting?

A.) 44    B.) 42    C.) 18    D.) 56    E.) NG

Tiebreaker:

When it is not raining, the probability that the Tigers will win a football game is $\frac{7}{10}$. When it is raining, the probability that the Tigers win is $\frac{2}{5}$. The probability of rain on any day in November is $\frac{1}{2}$. If the Tigers won the game they played on November 3, what is the probability that it was raining on that day?

Algebra II

ALGEBRA 2

1. B
2. C
3. B
4. A
5. B
6. A
7. C
8. D
9. D
10. A
11. B
12. D
13. B
14. B
15. A

Tiebreaker

$$\frac{4}{11}$$