

ECML
ALGEBRA I
2019

With Solutions

Essex County Math League
May 22, 2019
Algebra 1

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY only use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

- 1) Simplify: $\frac{1^{5m+2n}}{m^{-1}+n^{-1}}$, given $m=2$ and $n=3$.
A) $19/5$ B) $12/5$ C) $6/5$ D) $5/6$ E) NG
- 2) If $abc \neq 0$, the $\frac{a^3bc+ab^2c+abc^2}{abc} =$
A) $3abc$ B) $a^2 + b + c$ C) $a + b + c$ D) $\frac{a^3+b^2+c^2}{abc}$ E) NG
- 3) The number of real values of x which satisfy the equation $2^{2x^2-7x+4} = 1$ is:
A) 0 B) 1 C) 2 D) 4 E) NG
- 4) Given the following equations
I $2x + y = 3$ II $x + 3y = 4$ III $y = \frac{1}{3}x - 1$ IV $\frac{1}{3}y = -x + 1$ V $y - 3x = 2$
Which pair of equations represent perpendicular lines?
A) I, II B) III, V C) II, III D) II, V E) NG
- 5) Evaluate the expression: $\frac{P+Q}{P-Q} - \frac{P-Q}{P+Q}$, given $P = x + y$ and $Q = x - y$
A) $\frac{x^2-y^2}{xy}$ B) $\frac{x^2-y^2}{2xy}$ C) 1 D) $\frac{x^2+y^2}{xy}$ E) NG

- 6) If $(a + 3)(a + 5) - (a - 4)(a - 2) = 0$, then $a =$
- A) -2 B) $-1/2$ C) 0 D) $1/2$ E) NG
- 7) What speed must a motorist average, in km/hr, on the return trip from a city 100 km from home in order to have an average speed of 32 km/hr for the round trip, if the average speed going out is 20 km/hr?
- A) 40 B) 60 C) 64 D) 76 E) NG
- 8) A number increased by one-fifth of its additive inverse is equal to 8. Find the number.
- A) $20/3$ B) $32/5$ C) 10 D) $39/5$ E) NG
- 9) If $x \# y = 2x + y$, then $(2 \# ((3 \# 1) \# 2))$ equals...
- A) 8 B) 11 C) 13 D) 20 E) NG
- 10) Factor completely: $x^2 - (3a + 4b)x + 12ab$
- A) prime B) $(x - 3a)(x + 4b)$ C) $(x - 3a)(x - 4b)$ D) $(x + 3a)(x + 4b)$ E) NG
- 11) Simplify: $\frac{x-y}{\sqrt{x}-\sqrt{y}}$, given $x \neq 0$ and $y \neq 0$.
- A) $\frac{\sqrt{x}-\sqrt{y}}{x-y}$ B) $\frac{\sqrt{x}+\sqrt{y}}{x-y}$ C) $\sqrt{x} + \sqrt{y}$ D) $\sqrt{x} - \sqrt{y}$ E) NG
- 12) If the points $(-1, -2)$, $(3, 7)$, and $(11, y)$ are collinear, then $y =$
- A) 18 B) 25 C) 11 D) 15 E) NG
- 13) Find the LCM of $x^2 - 15x + 36$ and $x^3 - 3x^2 - 2x + 6$
- A) $x-3$ B) $(x - 3)(x - 12)(x^2 + 2)$ C) $(x - 3)(x - 12)(x^2 - 2)$
- D) $(x - 3)(x - 12)(x + 2)(x - 2)$ E) NG
- 14) Express the product as a single radical (surd): $\sqrt[3]{a^2} \times \sqrt{a^3}$
- A) a B) $\sqrt[5]{a^6}$ C) $\sqrt[6]{a^5}$ D) $\sqrt[6]{a^{13}}$ E) NG

- 15) Given the linear equation $\frac{x}{2} - \frac{y}{3} = 1$, find the sum of the x and y intercepts.
A) -16 B) -1 C) 5 D) 6 E) NG
- 16) Find three consecutive integers whose sum is equal to the product of the first two.
A) 5, 6, 7 B) 2, 3, 4 C) 4, 5, 6 D) 3, 4, 5 E) NG
- 17) Find the sum of the coordinates of the vertex of the parabola: $y = x^2 + 4x - 5$
A) -11 B) -7 C) 7 D) 11 E) NG
- 18) Find the ratio of a to b if, $\frac{2a-b}{2a} = \frac{-3}{4}$
A) 7/2 B) 2/7 C) -2/7 D) -7/2 E) NG
- 19) If $y = x + 1$, what is the value of: $|x - y| + |y - x|$
A) -2 B) 0 C) 2 D) 4 E) NG
- 20) If the ordered pair (a, b) is the solution of the following system of equations, what is the value of $a - b$. $3a - 2b = 16$ and $2a - 3b = 14$
A) 5 B) 6 C) 7 D) 8 E) NG

Tie breaker: This question will be scored only if there is a tie amongst the highest scorers.
Please write your answer in the area described by the proctors.

Two glasses of the same size contain pure orange juice. One is $\frac{1}{2}$ full and the other is $\frac{1}{3}$ full. Both are filled to the top with water and poured into a large container and mixed. Half of the mixture is poured back into one of the two glasses. What part of this glass is pure orange juice?

Answers to 2019 ECML Contests

Algebra 1

- 1) C
- 2) B
- 3) C
- 4) D
- 5) A
- 6) B
- 7) E
- 8) C
- 9) D
- 10) C
- 11) C
- 12) B
- 13) C
- 14) D
- 15) B
- 16) D
- 17) A
- 18) B
- 19) C
- 20) B

TB $5/12$

Algebra 2

- 1) C
- 2) A
- 3) D
- 4) C
- 5) C
- 6) E
- 7) D
- 8) A
- 9) C
- 10) C
- 11) B
- 12) D
- 13) B
- 14) C
- 15) D
- 16) E
- 17) C
- 18) C
- 19) A
- 20) A

TB $\frac{\sqrt{2}}{2}$

Advanced Math

- 1) E
- 2) C
- 3) A
- 4) B
- 5) D
- 6) D
- 7) C
- 8) A
- 9) A
- 10) A
- 11) B
- 12) B
- 13) D
- 14) B
- 15) A

TB $y = \pm 2$