

Essex County Math League

May 24, 2023

ADVANCED MATH

DIRECTIONS: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

- 1) Given $f(x) = x + 2$ and $g(x) = x^2 - 2x - 10$, find the sum of the solution(s) to $f(g(x)) = 0$.
- A) -2
B) 0
C) 2
D) 6
E) NG
- 2) When defined, $\csc(x) + \tan(x)$ is equal to which of the following:
- A) $\frac{1+\sin(x)}{\cos(x)}$
B) $\frac{1+\cos(x)-\cos^2(x)}{\sin(x)\cos(x)}$
C) $\cot(x)$
D) $\frac{1-\cos(x)}{\cos(x)\sin(x)}$
E) NG
- 3) Let $f(x) = \frac{x+4}{3x}$, then $f^{-1}(x-2) = \underline{\hspace{2cm}}$.
- A) $\frac{4}{3x-1}$
B) $\frac{4}{3x-7}$
C) $\frac{4x}{3x-1}$
D) $\frac{4x}{3x-5}$
E) NG
- 4) Find the sum of all the solutions to the equation: $2\sin^2(x) - \sin(x) = 0$ on the closed interval $[0, 2\pi]$.
- A) $\frac{\pi}{6}$
B) $\frac{7\pi}{6}$
C) 2π
D) 4π
E) NG

- 5) Suppose $\log_4 y = \frac{3}{2} - x$ and $y = 8^{-x+1}$, find the value of y .
- A) 0
B) $\frac{3}{2}$
C) 4
D) 8
e) NG
- 6) Evaluate the following infinite sum: $S = \frac{1}{4} - 3\left(\frac{1}{4}\right)^2 + 3^2\left(\frac{1}{4}\right)^3 - 3^3\left(\frac{1}{4}\right)^4 + \dots$
- A) $\frac{1}{7}$
B) $\frac{-3}{4}$
C) 1
D) ∞
E) NG
- 7) Suppose 4 and $3 + 2i$ are two roots of a third-degree polynomial with integer coefficients of the form $ax^3 + bx^2 + cx + d$, What is the sum of $a + b + c + d$?
- A) 10
B) -23
C) -24
D) -52
E) NG
- 8) A sine curve with equation $y = a \sin(bx + c) + d$ has one maximum point (3, 10) and one minimum point at (7, -1) on the interval [0, 10]. Find the value of $abcd$?
- A) -24
B) $\frac{-3\pi}{2}$
C) $\frac{3\pi}{2}$
D) 24
E) NG

- 9) Given $\triangle ABC$, the Law of Tangents is: $\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}$. If $\angle A = 40^\circ$, $\angle B = 10^\circ$, and $b = 10$, find the length of side a to the nearest tenth.
- A) 36.6
B) 37.1
C) 37.5
D) 37.9
E) NG
- 10) The probability that a tetrahedron die will roll a six face down is $\frac{1}{4}$. What is the probability that exactly 4 out of 10 tosses will show a six?
- A) .250
B) .146
C) .0250
D) .0146
E) NG
- 11) If $f(x) = -(x - 2)^2 + 3$ is translated three units to the left and two units down to form $g(x)$. What is the sum of the values of x if $x = g^{-1}(0)$?
- A) -2
B) -1
C) 0
D) 3
E) NG
- 12) If $\pi^m = e^n$, what is the ratio of $m:n$?
- A) $\frac{e}{\pi}$
B) $\frac{\pi}{e}$
C) $\ln(\pi)$
D) $\frac{1}{\ln(\pi)}$
E) NG

13) A circle with equation $x^2 + y^2 - 6x + 16y = k$, contains the point $(3, 0)$. What is the circumference of the circle?

- A) 4π
- B) 6π
- C) 9π
- D) 12π
- E) NG

14) Given: $\lim_{x \rightarrow 0} \frac{\sqrt{x+6} - \sqrt{6}}{x} = \frac{\sqrt{a}}{b}$, what is the sum $a + b$?

- A) 8
- B) 12
- C) 18
- D) The limit does not exist
- E) NG

15) Given the ellipse, find the sum of the lengths of the major and minor axis.

$$\frac{(x + 4)^2}{9} + \frac{(y - 2)^2}{16} = 1$$

- A) 5
- B) 7
- C) 14
- D) 25
- E) NG

Tie Breaker: *This question must be written on the scantron sheet in the area indicated by the proctors. This question will only be scored to break a tie between the highest scorers on the contest.*

The bacteria on a school desk double every 20 minutes. At 8am there are 10 bacteria on the desk. At 12 noon, a student wipes down the desk and kills $\frac{1}{2}$ the bacteria present. How many bacteria are on the desk at 3:20 pm?

ANSWER KEY

1. C
2. B
3. B
4. D
5. D
6. A
7. C
8. E
9. B
10. B
11. A
12. D
13. E
14. C
15. C

TB. There are two possible answers depending on the interpretation.

- 20,971,520 or 5×2^{22}
- 83,886,080 or 10×2^{23}