Essex County Math League

Wednesday, May 25, 2022

Advanced Math

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Advanced Math

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers; incorrect answers will NOT lower your score. You may ONLY use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is NG which means "Not Given" and is a valid answer that indicates that the correct answer is not among the answers given.

1. Given $f(x) = \frac{1}{x^3}$, find $\frac{f(x+h)-f(x)}{h}$

A.) $\frac{-3x^2+3xh+h^2}{x^3(x+h)^3}, h \neq 0$ B.) $\frac{h^2}{x^3(x+h)^3}, h \neq 0$ C.) $\frac{-3h^2}{x^3(x+h)^3}, h \neq 0$

D.) $\frac{-3x^2-3xh-h^2}{x^3(x+h)^3}, h \neq 0$ E.) NG

2. Simplify: $\frac{6-x}{x(x+2)} + \frac{x+2}{x^2} + \frac{16}{x^2(x+2)}$

A.) $\frac{2(3x+10)}{x^2(x+2)}$ B.) $\frac{10}{x^2}, x \neq -2$ C.) $\frac{4(2x+5)}{x^2(x+2)}$ D.) $\frac{24}{x^3+3x^2+2x}$ E.) NG

3. $\sin\left(\cos^{-1}\frac{3}{x}\right) =$

A.) $\frac{\pi}{2} - \frac{3}{x}$ B.) $\pm\frac{\sqrt{x^2-9}}{x}$ C.) $\frac{\sqrt{x^2-9}}{x}$ D.) $\frac{\sqrt{x^2-9}}{|x|}$ E.) NG

4. $\cos^3\theta + \sin^3\theta =$

A.) $(\cos\theta + \sin\theta)^3$
B.) $(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)^2$
C.)$(\cos\theta + \sin\theta)\left(1 - \frac{1}{2}\sin 2\theta\right)$
D.) $\cos\theta + \sin\theta$
E.) NG

5. Simplify: $x^2(x^2+1)^{-5} - (x^2+1)^{-4}$

A.) $x^2 - 1$ B.) $x^2 + 1$ C.) $\frac{1}{(x^2+1)^5}$ D.) $-\frac{1}{(x^2+1)^5}$ E.) NG

6. If $0 < a < 1$ and $0 < b < 1$, then solve the given equation **on the interval** $[0, 2\pi)$:

$$(\sin x + a)(\cos x - b) = 0$$

A.) $\sin^{-1}(-a), \cos^{-1} b$
B.) $2\pi - \sin^{-1}(-a), \pi - \sin^{-1}(-a), \cos^{-1} b, 2\pi - \cos^{-1} b$
C.) $\sin^{-1}(-a), \pi - \sin^{-1}(-a), \cos^{-1} b, 2\pi - \cos^{-1} b$
D.) $2\pi + \sin^{-1}(-a), \pi - \sin^{-1}(-a), \cos^{-1} b, 2\pi - \cos^{-1} b$
E.) NG

7.

$$\sum_{k=3}^{n}\left(\sum_{j=5}^{a} 4\right)$$

A.) $4an$ B.) $4(n-3)(a-5)$ C.) $4(n-2)(a-4)$

D.) $4ajnk$ E.) NG

8. If $\frac{3\pi}{2} < \theta < 2\pi$, and $\cos\theta = a$, then $\cos\left(\frac{\theta}{2}\right) =$

A.) $\sqrt{\frac{1+a}{2}}$ B.) $-\sqrt{\frac{1+a}{2}}$ C.) $\sqrt{\frac{1-a}{2}}$ D.) $-\sqrt{\frac{1-a}{2}}$ E.) NG

9. Simplify:

$$\frac{(x+2)^{\frac{3}{4}}(x+3)^{-\frac{2}{3}} - (x+3)^{\frac{1}{3}}(x+2)^{-\frac{1}{4}}}{\left((x+2)^{\frac{3}{4}}\right)^2}$$

A.) $\frac{1}{(x+2)^{\frac{7}{4}}(x+3)^{\frac{2}{3}}}$ B.) $-\frac{1}{(x+2)^{\frac{7}{4}}(x+3)^{\frac{2}{3}}}$ C.) $\frac{5}{(x+2)^{\frac{7}{4}}(x+3)^{\frac{2}{3}}}$

D.) $\frac{(x+3)^{\frac{1}{3}}\left((x+3)^{\frac{7}{3}} - (x+2)^{\frac{13}{4}}\right)}{(x+2)^{\frac{3}{4}}}$ E.) NG

10. Find the directrix of the parabola that has focus $(-1, 7)$, opens left, and passes through the point $(-6, 11)$.

A.) $x = -6 + \sqrt{41}$ B.) $y = 11 - \sqrt{41}$ C.) $x = -3$

D.) $y = 1 + \sqrt{41}$ E.) NG

11. As The Wizard of Odds drifts away from Sapphire City in his hot air balloon, Dorothea watches him leave. She notices that he is traveling at a constant altitude and at a speed of 20 miles per hour. She wonders how far he is above the ground. At one instant, she notes that the angle of elevation to the Wizard's balloon is 33°. Three minutes later, the angle of elevation to the balloon is now 17°. Using the information provided, calculate the altitude of the balloon.

A.) 2.910 mi B.) 0.890 mi C.) 0.578 mi D.) 1.890 mi E.) NG

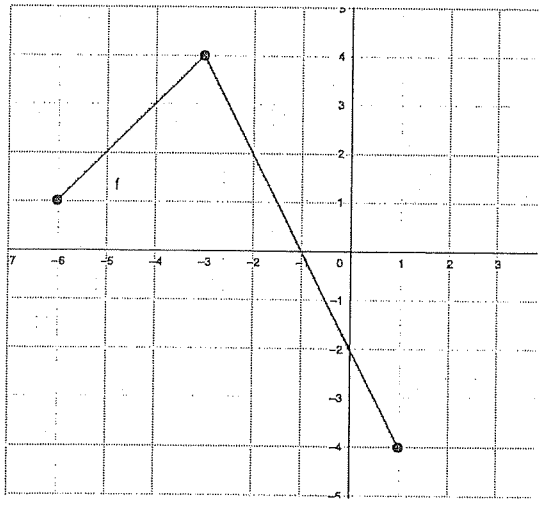
12. A tornado siren is heard by Isaac and Gottfried who are standing 3500 feet apart. Isaac hears the siren two seconds before Gottfried. Assume that sound travels at 1100 feet per second and that Isaac and Gottfried are standing on the x-axis with the midpoint between them at the origin. Write an equation for the curve on which the tornado siren must be located.

A.) $\frac{x^2}{1210000} + \frac{y^2}{1852500} = 1$ B.) $\frac{x^2}{3062500} - \frac{y^2}{1210000} = 1$ C.) $\frac{x^2}{1210000} - \frac{y^2}{1852500} = 1$

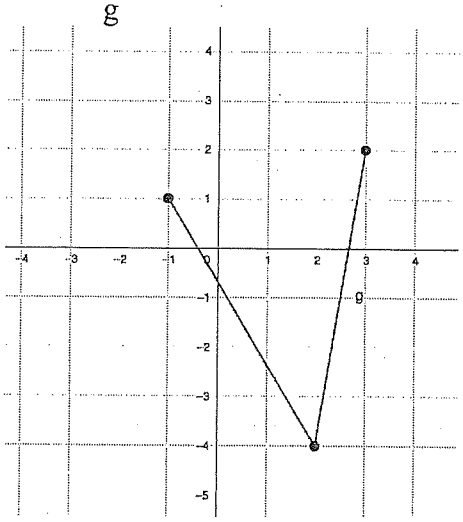
D.) $\frac{x^2}{3062500} + \frac{y^2}{1210000} = 1$ E.) NG

13. Given the graphs of f and g, state the domain of $g(f(x))$.

f



g



A.) $[-4, 4]$ B.) $[-6, 1]$ C.) $[-1, 3]$

D.) $[-6, -4] \cup [-\frac{5}{2}, -\frac{1}{2}]$ E.) NG

14. Given that a, b, c, d, and e are all positive constants, which of the following is equivalent to

$$(\log_a b^2)(log_{c^2} a)$$

A.) $\log_c b$ B.) $\frac{\log 2b}{\log 2c}$ C.) $\log\left(\frac{b}{c}\right)$

D.) All of the above E.) NG

15. The second hand on a particular clock is 4 inches long. The tip of the second hand is traveling at 26 inches per minute. Is this clock fast or slow? If this clock is placed next to an accurate clock at 12:00 pm, what time will it read (to the nearest second) when the accurate clock shows 12:00 pm on the next day?

A.) This clock is accurate; it will also read 12:00:00.
B.) This clock is slow; it will read 11:10:19.
C.) This clock is fast; it will read 12:49:41.
D.) There is not enough information given to determine whether the clock is fast or slow.
E.) NG

Tiebreaker

Five personalized letters and envelopes are addressed to five different people. The letters are randomly inserted into the envelopes. What is the probability that exactly one letter will be inserted into the correct envelope?

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1. D
2. B
3. D
4. C
5. D
6. D
7. C
8. B
9. B
10. A
11. C
12. C
13. D
14. A
15. C

Tiebreaker

$\frac{3}{8}$