# Essex County Math League
# May 22, 2019
# Advanced Math

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY only use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

1) Which ordered pair represents the center of the given circle:
$x^2 + y^2 - 6x + 3y - 12 = 0$

A) (9, 4) B) (-3, 2) C) (3, -2) D) (-6, 4) E) NG

2) T varies directly as the square of R and inversely as the cube of S. If R is tripled and S is doubled, then the value of T is:

A) multiplied by 3/2 B) multiplied by 6 C) multiplied by 9/8
D) multiplied by 2 E) NG

3) If $log_{10}(x) = y$ *and* $log_e(10) = \frac{1}{m}$ then which of the following is true?

A) $\ln(x) = \frac{y}{m}$ B) $\ln(x) = \frac{m}{y}$ C) $\ln(y) = \frac{m}{x}$ D) $\ln(y) = \frac{x}{m}$ E) NG

4) Given x<y, evaluate $\arcsin\left(\frac{x}{y}\right) + \arccos\left(\frac{x}{y}\right) =$

A) 0 B) $\frac{\pi}{2}$ C) $\frac{\pi}{4}$ D) $\pi$ E) NG

5) If $f(x) = \sqrt{x-1}$, then $f^{-1}(x+1) =$

A) $x^2$ B) $x^2+1$ C) $x^2+2x+1$ D) $x^2+2x+2$ E) NG

6) Where defined, $\sec(x) + \tan(x) =$

A) $\frac{\sin(x)}{1+\cos(x)}$ B) $\frac{\sin(x)}{1-\cos(x)}$ C) $\frac{\cos(x)}{1+\sin(x)}$ D) $\frac{\cos(x)}{1-\sin(x)}$ E) NG

7) A weighted coin comes up heads 60% of the time when tossed. Rounded to the nearest thousandth, what is the probability that the coin comes up heads at least twice when tossed 8 times?

A) 0.894 B) 0.950 C) 0.991 D) 0.999 E) NG

8) Find the sum of all the solution(s) to the equation $2\cos^2(x) - \cos(x) - 1 = 0$ in the interval $[0, 3\pi]$.

A) $\frac{20\pi}{3}$ B) $\frac{41\pi}{6}$ C) $\frac{25\pi}{3}$ D) $\frac{26\pi}{3}$ E) NG

9) Evaluate the sum: $S = \left(\frac{1}{5}\right) - 3\left(\frac{1}{5}\right)^2 + 9\left(\frac{1}{5}\right)^3 - 27\left(\frac{1}{5}\right)^4 + \cdots$

A) $\frac{1}{8}$ B) $\frac{1}{2}$ C) $\frac{5}{8}$ D) $\frac{5}{2}$ E) NG

10) $\cos\left(2\arcsin\left(\frac{12}{13}\right)\right) =$

A) $\frac{-119}{169}$ B) $\frac{5}{13}$ C) $\frac{120}{169}$ D) $\frac{10}{13}$ E) NG

11) Find the polynomial of lowest degree with integral coefficients whose roots are 6 and $4 - 3i$.

A) $x^3 - 10x^2 - 18x - 12 = 0$ B) $x^3 - 14x^2 + 73x - 150 = 0$
C) $4x^3 - 24x^2 + 150 = 0$ D) $16x^3 - 24x^2 + 4x - 150 = 0$ E) NG

12) The sum of a number and its reciprocal is 5. What is the sum of the cube of the number and the cube's reciprocal?

A) 105 B) 110 C) 115 D) 125 E) NG

13) A point has rectangular coordinates (3, 4). The polar coordinates are $(5, \theta)$. What is the value of $\theta$ to the nearest degree?

A) 37° B) 30° C) 51° D) 53° E) NG

14) The probability that the Mets will win their division is p. The probability that the Yankees will win their division, a different division than the Mets, is q. What is the probability that only 1 team will win its division?

A) pq B) p + q – 2pq C) |p - q| D) 1 – pq E) NG

15) A sine curve with equation $y = a\sin(bx + c) + d$ has one maximum point at (2, 7) and one minimum point at (8, -1) on the interval [0. 10]. What is the value of $a^2b$ ?

A) $\frac{8\pi}{3}$ B) $\frac{16\pi}{3}$ C) $6\pi$ D) $\frac{32\pi}{3}$ e) NG

Tie breaker: This question will be scored only if there is a tie amongst the highest scorers. Please write your answer in the area described by the proctors.

List all the asymptotes of the graph of $y = \frac{\sqrt{4x^2+12x-40}}{x+3}$

Answers to 2019 ECML Contests

| Algebra 1 | | Algebra 2 | | Advanced Math | |
|---|---|---|---|---|---|
| 1) | C | 1) | C | 1) | E |
| 2) | B | 2) | A | 2) | C |
| 3) | C | 3) | D | 3) | A |
| 4) | D | 4) | C | 4) | B |
| 5) | A | 5) | C | 5) | D |
| 6) | B | 6) | E | 6) | D |
| 7) | E | 7) | D | 7) | C |
| 8) | C | 8) | A | 8) | A |
| 9) | D | 9) | C | 9) | A |
| 10) | C | 10) | C | 10) | A |
| 11) | C | 11) | B | 11) | B |
| 12) | B | 12) | D | 12) | B |
| 13) | C | 13) | B | 13) | D |
| 14) | D | 14) | C | 14) | B |
| 15) | B | 15) | D | 15) | A |
| 16) | D | 16) | E | | |
| 17) | A | 17) | C | | |
| 18) | B | 18) | C | | |
| 19) | C | 19) | A | | |
| 20) | B | 20) | A | | |
| TB | 5/12 | TB | $\frac{\sqrt{2}}{2}$ | TB | $y = \pm 2$ |