# Essex County Math League
## May 24, 2018
## Advanced Math

Directions: You may write on this test. Be sure that your name, subject, and school (including town name) are on the answer sheet. Mark the answer sheet with dark, careful marks using a #2 pencil. Your score will be determined by your number of correct answers, incorrect answers will NOT lower your score. You MAY only use a calculator on this test that is approved for use on the SAT's. The answer to the tie-breaker should be placed on the answer sheet in the place indicated by the proctors. The tie-breaker will be scored only in the case of a tie between the top scorers, and will not count as part of the team score. The fifth choice for each question is, NG, which means, "not given" and is a valid answer that indicates that the correct answer is not among the answers given.

1) Find the value of: $\sin(\frac{3\pi}{4}+x)+\sin(\frac{3\pi}{4}-x)=$

a) $\sqrt{2}\sin(x)$   b)   -1   c) $\sqrt{3}\sin(x)$ d) $\sqrt{2}\cos(x)$ e)   NG

2) Find: $Lim_{x\to\infty}\dfrac{x^2-x+1}{x^3-3}=$

a)   0   b)   2/3   c)   3/2   d)   $\infty$   e)   NG

3) If $\log_{10}(x)=y$ and $\ln(10)=\dfrac{1}{m}$, then which of the following is true?

a) $\ln(x)=\dfrac{y}{m}$   b) $\ln(x)=\dfrac{m}{y}$   c) $\ln(y)=\dfrac{m}{x}$   d) $\ln(y)=\dfrac{x}{m}$   e)   NG

4) Evaluate: $\arcsin(3/5)+\arccos(3/5)=$

a)   0   b)   $\dfrac{\pi}{2}$   c)   $\dfrac{\pi}{4}$   d)   $\pi$   e)   NG

5) Given: $f(x)=\sqrt{x-1}$, then $f^{-1}(x+1)=$

a) $x^2$ b) $x^2+1$   c) $x^2+2x+1$   d) $x^2+2x+2$ e)   NG

6) Where defined, $\sec(x) + \tan(x)$ is equal to which of the following?

a) $\dfrac{\sin(x)}{1+\cos(x)}$    b) $\dfrac{\sin(x)}{1-\cos(x)}$    c) $\dfrac{\cos(x)}{1+\sin(x)}$    d) $\dfrac{\cos(x)}{1-\sin(x)}$    e)    NG

7) Let $f(x) = \dfrac{x+3}{2x}$, then $f^{-1}(x+4) =$

a) $\dfrac{3}{2x-1}$    b) $\dfrac{3}{2x+3}$ c)    $\dfrac{3}{2x+7}$    d) $\dfrac{3}{2x+8}$ e)    NG

8) A weighted coin comes up heads 60% of the time when tossed. The coin is tossed 8 times. What is the probability that the coin comes up heads at least twice? (round your answer to the nearest thousandth.)

a)    0.894    b)    0.950    c)    0.991    d)    0.999    e)    NG

9) Find the sum of all the solutions(s) to the following equation on the interval $[0,3\pi]$:
$$2\cos^2(x) - \cos(x) - 1 = 0$$

a) $\dfrac{20\pi}{3}$    b) $\dfrac{41\pi}{6}$    c) $\dfrac{25\pi}{3}$    d) $\dfrac{26\pi}{3}$    e)    NG

10) Evaluate the following infinite sum: $\dfrac{1}{5} - 3(\dfrac{1}{5})^2 + 9(\dfrac{1}{5})^3 - 27(\dfrac{1}{5})^4 \ldots$

a)    1/8    b)    ½    c)    5/8    d)    5/2    e)    NG

11) Given $f(x) = \sin(2x-1)$ and $f(g(x)) = x^3 + 1$, then g(x) =?

a) $\dfrac{1}{2}\left[\sin^{-1}(x^3+1)+1\right]$    b) $\sqrt[3]{\sin(2x-1)-1}$    c) $\sqrt[3]{\sin^{-1}(2x-1)-1}$

d)    $\sin^3(2x-1)+1\,\sin^3(2x-1)+1$    e)    NG

12) Let $f(x)=x^{10}-17x^6+7x^3+12$. Find the remainder when f(x) is divided by x – 2.

    a)    -108   b)    4    c)   6    d)    20    e)    NG

13) If $5^{2x+y}=0.2$, and $25^{2x-y}=5$, then the value of x + y =

    a) $-\dfrac{5}{4}$   b)    $-\dfrac{7}{8}$   c)   $\dfrac{1}{4}$   d)    $\dfrac{5}{8}$   e)    NG

14) Which of the following are asymptotes for: $f(x)=\dfrac{2x^2+5x-3}{x^2-9}$

    I)    x = 3     II)    x = -3     III)   y = 2

    a) I and II only   b) I, II, and III   c) I and III only   d) II and III only  e) NG

15) What is the sum of the solution(s) of the equation: $\log_3(x)+\log_3(x-4)=\log_3(x+6)$ ?

    a)    -6   b)    -5   c)    5   d)    10   e)    NG

Tie breaker: This question will be scored only if there is a tie amongst the highest scorers. Please write your answer in the area described by the proctors.

A cosine curve has two successive maximum points at (2, 3) and (8, 3), and a minimum point at (5, -5). Write the exact sum of the numerical values of the horizontal and vertical shifts for the curve.

| ECML Answers 2018 | | | | | | | |
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| Algebra 1 | | Geometry | | Algebra 2 | | Advanced Math | |
| Quest. # | Answer | Quest. # | Answer | Quest. # | Answer | Quest. # | Answer |
| 1) | C | 1) | B | 1) | C | 1) | D |
| 2) | E | 2) | A | 2) | D | 2) | A |
| 3) | C | 3) | C | 3) | C | 3) | A |
| 4) | B | 4) | B | 4) | E | 4) | B |
| 5) | D | 5) | A | 5) | A | 5) | D |
| 6) | B | 6) | C | 6) | A | 6) | D |
| 7) | C | 7) | C | 7) | B | 7) | C |
| 8) | C | 8) | B | 8) | A | 8) | C |
| 9) | D | 9) | D | 9) | A | 9) | A |
| 10) | A | 10) | C | 10) | D | 10) | A |
| 11) | A | 11) | C | 11) | C | 11) | A |
| 12) | C | 12) | D | 12) | C | 12) | B |
| 13) | E | 13) | D | 13) | C | 13) | B |
| 14) | A | 14) | A | 14) | B | 14) | C |
| 15) | D | 15) | B | 15) | C | 15) | E |
| 16) | B | 16) | B | 16) | C | TB | 2+(1-)=1 |
| 17) | B | 17) | B | 17) | D | | |
| 18) | E | 18) | A | 18) | E | | |
| 19) | A | 19) | C | 19) | C | | |
| 20) | A | 20) | C | 20) | D | | |
| TB | (1, 8) | TB | "8 | TB | "1/16" | n | |